Force-free magnetic fields in the solar atmosphere

N. Seehafer^{1,*}, M. Fuhrmann¹, G. Valori², and B. Kliem^{2,3}

¹ Institut für Physik, Universität Potsdam, Am Neuen Palais 10, D-14469 Potsdam, Germany

² Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany

³ Kiepenheuer-Institut für Sonnenphysik, Schöneckstraße 6, D-79104 Freiburg, Germany

Received 2007 Aug 30, accepted 2007 Oct 17 Published online 2007 Dec 15

Key words Sun: activity - Sun: atmosphere - Sun: magnetic fields

Reliable measurements of the solar magnetic field are restricted to the level of the photosphere. For about half a century attempts have been made to calculate the field in the layers above the photosphere, i.e. in the chromosphere and in the corona, from the measured photospheric field. The procedure is known as magnetic field extrapolation. In the superphotospheric parts of active regions the magnetic field is approximately force-free, i.e. electric currents are aligned with the magnetic field. The practical application to solar active regions has been largely confined to constant- α or linear force-free fields, with a spatially constant ratio, α , between the electric current and the magnetic field. We review results obtained from extrapolations with constant- α force-free fields, in particular on magnetic topologies favourable for flares and on magnetic and current helicities. Presently, different methods are being developed to calculate non-constant- α or nonlinear force-free fields from photospheric vector magnetograms. We also briefly discuss these methods and present a comparison of a linear and a nonlinear force-free magnetic field extrapolation applied to the same photospheric boundary data.

© 2007 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

1 Introduction

Magnetic fields play a key role in solar physics and in solar activity in particular. To understand the physical mechanism of any of the activity phenomena observable in the solar atmosphere one needs to know the underlying magnetic field. The magnetic field also provides the link between different manifestations of solar activity like, for instance, sunspots, flares, or coronal mass ejections. Therefore, there is a strong need for information about the magnetic vector throughout the atmosphere. Unfortunately, reliable magnetic field measurements are still restricted to the level of the photosphere, where the inverse Zeeman effect in Fraunhofer lines is observable. The situation here is improving only very slowly due to elementary difficulties in unambiguously deriving the magnetic field from polarimetric measurements in chromospheric or coronal spectral lines. As an alternative to measurements in these superphotospheric layers, for about half a century attempts have been made to calculate the field there from the measured photospheric field using physically plausible assumptions. The procedure is known as magnetic field extrapolation.

For typical plasma parameters in the superphotospheric parts of active regions, except for times of explosive events, the magnetic energy density dominates over the thermal, kinetic and gravitational energy densities. This implies that, if appreciable currents are present, these must be aligned with the magnetic field, since otherwise the resulting Lorentz forces could not be balanced by the nonmagnetic forces.

^{*} Corresponding author: seehafer@uni-potsdam.de



Thus, the magnetic field must be approximately force-free, characterized by the equations

$$\nabla \times \boldsymbol{B} = \alpha(\boldsymbol{r}) \, \boldsymbol{B} \,, \tag{1}$$

$$\nabla \cdot \boldsymbol{B} = 0, \qquad (2)$$

where $\alpha(r)$ denotes a scalar function of position r which, because of Eq. (2), is constant along the magnetic field lines. The approximation of force-freeness is presumably valid from the upper chromosphere up to heights of $\sim 1 R_{\odot}$ above the level of the photosphere in the corona (see Gary 2001, for a careful study of the height variation of the plasma β , i.e. the ratio between the thermal and magnetic pressures or energy densities). Extrapolation methods have been developed for different types of force-free fields:

- Potential fields. These correspond to the case $\alpha = 0$. The magnetic field **B** above the photosphere is considered as a vacuum or current-free or potential field, satisfying $\nabla \times \mathbf{B} = \mathbf{0}$. Potential field models were devised both for the field above limited photospheric areas, in particular active regions (Schmidt 1964; Teuber et al. 1977), and for the global field above the spherical photosphere (Altschuler & Newkirk 1969; Schatten et al. 1969).
- Linear force-free fields. These correspond to the case of a spatially constant (in general non-vanishing) α . Their determination leads to boundary value problems for a linear partial differential equation. Solutions were given by Nakagawa & Raadu (1972), Chiu & Hilton (1977), Seehafer (1978), Alissandrakis (1981), Semel (1988); see also reviews by Seehafer & Staude (1983), Gary (1989) and Sakurai (1989).

Nonlinear force-free fields. Here α is allowed to vary spatially. Unlike the extrapolation methods for current-free and constant-α force-free fields, which require only line-of-sight magnetograms, the non-constant-α force-free fields are calculated from photospheric vector magnetograms. Presently great efforts are made to refine the methods used here and to improve their applicability. Discussions and comparisons of the nonlinear extrapolation methods are found in Aly (1989), Sakurai (1989), McClymont et al. (1997), Schrijver et al. (2006) and Metcalf et al. (2007).

2 Helicity

The extrapolations with constant- α force-free fields, as well as those with current-free fields, have provided new insights into the physics of the activity phenomena. Some justification for the use of constant- α force-free fields comes from the hypothesis of Taylor (1974, 1986) relaxation. According to this hypothesis, a plasma with a high (but finite) electrical conductivity in which the internal energy is negligible compared to the magnetic energy relaxes from an arbitrary turbulent initial state to a force-free state with a spatially constant α , which follows from the assumption that the total magnetic helicity

$$H = \int_{V} \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}V,\tag{3}$$

where A is a magnetic vector potential, is conserved during relaxation to a state of minimum magnetic energy. This refers to the case where there is no flow of magnetic helicity through the boundary of the considered volume V. In solar active regions, there will be a competition between the injection of magnetic helicity through the photosphere and the Taylor relaxation towards constant- α force-free states.

The densities of magnetic and current helicity are defined by

$$h_{\rm m} = \boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot (\nabla \times \boldsymbol{A}), \quad h_{\rm c} = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}), \quad (4)$$

where $h_{\rm c}$ is related to the factor α of the force-free field by

$$\alpha = h_{\rm c}/\boldsymbol{B}^2 \,. \tag{5}$$

It is one of the results of extrapolations using constant- α force-free fields that α and h_c are predominantly negative in the northern and positive in the southern hemisphere of the Sun (Seehafer 1990). Furthermore, if α is spatially constant, h_m and h_c have the same sign. This can be expected to be still approximately valid if α has a predominant sign within individual active regions, as indicated by the observations. Thus, the magnetic helicity density h_m is presumably also predominantly negative in the northern and positive in the southern hemisphere.

The observation of current helicity in the solar atmosphere opens a window to the interior of the Sun. Namely, in the dynamo region in the solar convection zone magnetic and velocity fluctuations are believed to lead to a mean electromotive force

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u} \times \boldsymbol{b} \rangle = \alpha_{ij}^{(\text{dyn})} \langle B \rangle_j + \beta_{ijk} \frac{\partial \langle B \rangle_j}{\partial x_k} + \dots, \qquad (6)$$

where the first term on the right corresponds to the α -effect of mean-field dynamo theory (Krause & Rädler 1980); here angular brackets denote averages and u and b are the fluctuating or turbulent parts of the velocity and magnetic field. The α -effect is connected with the current helicity of the fluctuations by the relation

$$\sum_{i,j} \alpha_{ij}^{(\text{dyn})} \langle B_i \rangle \langle B_j \rangle = -\eta \langle \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b}) \rangle$$
(7)

(e.g., Rädler & Seehafer 1990; Seehafer 1994, 1996), valid if the magnetic fluctuations are statistically homogeneous in space and time; η is the magnetic diffusivity. In the isotropic case, $\alpha^{(dyn)}$ is then a scalar given by

$$\alpha^{(\mathrm{dyn})} = -\frac{\eta}{\langle \boldsymbol{B} \rangle^2} \langle \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b}) \rangle, \qquad (8)$$

while Eq. (7) reduces to the approximate relation

$$\alpha_{\varphi\varphi} \langle B_{\varphi} \rangle^2 \approx -\eta \langle \boldsymbol{b} \cdot (\nabla \times \boldsymbol{b}) \rangle \tag{9}$$

if the toroidal component $\langle B_{\varphi} \rangle$ of the mean magnetic field is large compared to the other components, as is presumably the case for the Sun. According to Eq. (9), $\alpha_{\varphi\varphi}$ and $\langle h_c \rangle$ are opposite in sign, which is confirmed by realistic model calculations for these two quantities in the convection zone (Kuzanyan et al. 2006). A direct comparison with observations is possible if active regions are considered as fluctuations in the sense of mean-field dynamo theory.

3 Bald patches

The extrapolations with constant- α force-free fields have also provided new insights into the physics of the explosive phenomena in active regions. For instance, results were obtained on magnetic topologies favourable for flares. One example is the potential role of separatrix surfaces made up of magnetic field lines touching the photospheric boundary from above, see Fig. 1, for the formation of electric current sheets and the fast release of stored magnetic energy by magnetic reconnection (Seehafer 1986). The field lines of closed magnetic structures above the photosphere define a mapping from the photosphere to itself. This mapping is discontinuous if field lines end in a magnetic neutral point (where the field vanishes) - traditionally considered as a favourable site for magnetic reconnection - or touch the photosphere from above as illustrated in Fig. 1. These latter topological elements have been termed bald patches (Bungey et al. 1996; Titov & Démoulin 1999; Titov et al. 1993) because if field lines starting from photospheric grid points are displayed in an overview, the areas around segments of the neutral line of the photospheric normal component where the field lines touch the photosphere from above are apparently free of field lines – similarly as bald patches on a head are free of hairs.



Fig.1 Critical field line touching the photosphere from above. Discontinuities of B, for instance in the form of current sheets, may arise at the separatrix surfaces where the mapping from the photosphere to itself defined by the field lines is discontinuous.

4 Linear versus nonlinear extrapolation

Extrapolations with constant- α force free fields are

- able to give an overview of the field topology, also to identify important topological elements as, for example, bald patches,
- relatively simple,

but on the other hand

- they are not suitable for problems like strong localized currents and for detailed studies of the energy and helicity budgets of active regions,
- the calculated magnetic fields perhaps do not contain free energy, as needed, for instance, as the energy source for flares, since if they have come about by Taylor relaxation, they may represent states of minimum magnetic energy.

In the following we present a comparison of a linear with a nonlinear extrapolation applied to the same photospheric field values (in the linear case only the photospheric normal component was used). The boundary values were taken from a known force-free field constructed by Titov & Démoulin (1999) as a model for the field of an active region containing a current-carrying, i.e. twisted magnetic flux tube. The same field was used by Wiegelmann et al. (2006a) for testing a nonlinear force-free magnetic field extrapolation method different from the one used here, and we refer to their Table 1 for the parameters chosen for the field.

Field lines corresponding to the twisted loop of the Titov & Démoulin field are shown in the upper left panel of Fig. 2, along with their projections on the bottom and side faces of the rectangular box in which the solution was calculated. Red and green colour distinguish between field lines traced from the two photospheric areas where the loop ends. As is seen in the upper right panel of Fig. 2, these areas correspond to maxima of the modulus of $\alpha(\mathbf{r})$, which measures the field line twist, in the photospheric plane.

For our nonlinear extrapolation we used the method of Valori et al. (2005) (see also Valori et al. 2007). This belongs to the category of the relaxation methods. The magnetohydrodynamic equations are simulated in a simplified form with the equation for the fluid velocity containing a viscous dissipation term, but no external forcing terms, and the pressure term being neglected. Asymptotically in time, a quiescent state with a force-free magnetic field is reached. First a potential field satisfying the boundary condition for the photospheric normal component is calculated, in a next step the photospheric tangential components are overwritten by the prescribed values, and then the relaxation is started (the procedure is known as stress and relax method).

In the lower two panels of Fig. 2, results of the linear and nonlinear extrapolations are shown. The nonlinear extrapolation successfully reproduces the twisted loop (and also the rest of the field, not shown here), while the linear one clearly fails.

There are, however, still a number of problems with the nonlinear force-free magnetic field extrapolation methods. These include:

- Mathematical proofs for the existence of the solutions and convergence of the iterations used are still missing for most of the methods.
- The vector magnetograph data, in particular the transverse field component (perpendicular to the line of sight of the observer; this component can only be measured with an ambiguity of 180°), are noisy and not necessarily consistent with the assumption of force-freeness. To alleviate these problems, methods to appropriately preprocess the magnetograph data have been proposed (Wiegelmann et al. 2006b; Fuhrmann et al. 2007).
- There are no conditions for the lateral and upper boundaries (this applies to linear extrapolations as well).
- The calculations are numerically extensive.

5 Summary and outlook

- Magnetic field extrapolation is an indispensable tool for solar physics.
- Extrapolations using linear force-free fields (including current-free fields) are widely used presently. They can be very helpful if their limitations are borne in mind.
- The methods using nonlinear force-free fields have to be developed further to make them applicable routinely. This will be necessary in order to employ the large magnetographic data sets to be expected from satellite missions as well as from new ground-based telescopes in the near future.

References

Alissandrakis, C.E.: 1981, A&A 100, 197

Altschuler, M.D., Newkirk, Jr., G.: 1969, SoPh 9, 131

Aly, J.J.: 1989, SoPh 120, 19

Bungey, T.N., Titov, V.S., Priest, E.R.: 1996, A&A 308, 233

Chiu, Y.T., Hilton, H.H.: 1977, ApJ 212, 873

Fuhrmann, M., Seehafer, N., Valori, G.: 2007, A&A, in press

Gary, G.A.: 1989, ApJS 69, 323

Gary, G.A.: 2001, SoPh 203, 71



Fig. 2 (online colour at: www.an-journal.org) Field lines of the original Titov & Demoulin test field (upper left); $\alpha(x, y, z = 0)$ (upper right); field lines of the extrapolated linear force-free field for best-fitting $\alpha = 0.85$ (lower left); field lines of the extrapolated nonlinear force-free field (lower right).

- Krause, F., Rädler, K.-H.: 1980, *Mean-Field Magnetohydrodynamics and Dynamo Theory*, Akademie-Verlag, Berlin
- Kuzanyan, K.M., Pipin, V.V., Seehafer, N.: 2006, SoPh 233, 185
- McClymont, A.N., Jiao, L., Mikić, Z.: 1997, SoPh 174, 191
- Metcalf, T.R., DeRosa, M.L., Schrijver, C.J., et al.: 2007, SoPh, submitted
- Nakagawa, Y., Raadu, M.A.: 1972, SoPh 25, 127
- Rädler, K.-H., Seehafer, N.: 1990, in: H.K. Moffatt, A. Tsinober (eds.), *Topological Fluid Mechanics*, p. 157
- Sakurai, T.: 1989, SSRv 51, 11
- Schatten, K.H., Wilcox, J.M., Ness, N.F.: 1969, SoPh 6, 442
- Schmidt, H.U.: 1964, in: W.N. Hess (ed.), *Physics of Solar Flares*, NASA SP-50, p. 107
- Schrijver, C.J., DeRosa, M.L., Metcalf, T.R., et al.: 2006, SoPh 235, 161
- Seehafer, N.: 1978, SoPh 58, 215
- Seehafer, N.: 1986, SoPh 105, 223

- Seehafer, N.: 1990, SoPh 125, 219
- Seehafer, N.: 1994, EL 27, 353
- Seehafer, N.: 1996, PhRvE 53, 1283
- Seehafer, N., Staude, J.: 1983, Phys. Solariterr. 22, 5
- Semel, M.: 1988, A&A 198, 293
- Taylor, J.B.: 1974, PhRvL 33, 1139
- Taylor, J.B.: 1986, RvMP 58, 741
- Teuber, D., Tandberg-Hanssen, E., Hagyard, M.J.: 1977, SoPh 53, 97
- Titov, V.S., Démoulin, P.: 1999, A&A 351, 707
- Titov, V.S., Priest, E.R., Démoulin, P.: 1993, A&A 276, 564
- Valori, G., Kliem, B., Fuhrmann, M.: 2007, SoPh, in press
- Valori, G., Kliem, B., Keppens, R.: 2005, A&A 433, 335
- Wiegelmann, T., Inhester, B., Kliem, B., Valori, G., Neukirch, T.: 2006a, A&A 453, 737
- Wiegelmann, T., Inhester, B., Sakurai, T.: 2006b, SoPh 233, 215