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HELICITY AND THE SOLAR DYNAMO

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ABSTRACT

The central mechanism in traditional mean-field dynamo theory is the α-effect, and it has been found that the presence of kinetic or magnetic helicities is favourable for the effect, which corresponds to the simultaneous generation of magnetic helicities in the mean field and in the fluctuations, the generation rates being equal in magnitude and opposite in sign. Generally, the two helicities generated by the α-effect, that in the mean field and that in the fluctuations, have either to be dissipated in the generation region or to be transported out of this region. The latter presumably leads to the observed appearance of magnetic helicity in the solar atmosphere, which thus provides valuable information on dynamo processes inaccessible to in situ measurements. We have included details of two numerical dynamo studies in the present review, one for a "laminar" dynamo, where no averaging is applied, the other for a mean-field dynamo. In the first case the full nonlinear system of the incompressible MHD equations is studied in idealized rectangular geometry, with an external forcing of the Roberts type driving a flow in the form of an array of convection-like rolls. Defining mean fields by appropriate averages, it is found that there is a segregation of magnetic helicity between the mean field and the fluctuations similar to that predicted by the mean-field theory of the α-effect. The mean-field calculations are done in a quasi-linear approximation for the turbulence, for realistic spherical geometry, with compressibiliy included and using a profile of the solar internal rotation rate obtained from helioseismic inversions. The results are compared with observations, concentrating on the observational finding that the moduli of the averaged values of the force-free twist parameter α_{ff} and and the current helicity H_C increase from zero at the equator towards higher latitudes and attain a certain saturation level at middle latitudes (at about $20° - 30°$). On the assumption that the α-effect is operating in a thin spherical shell, the best coincidence between calculated and observed quantities is found for α-effect operation close to the bottom of the convection zone.

INTRODUCTION TO THE DYNAMO PROBLEM AND ITS CONNECTION WITH HELICITY

The dynamo for the global solar magnetic field is assumed to operate in the convection zone and to consist of the cyclic generation of a toroidal (azimuthal) field from a poloidal one (whose field lines lie in planes containing the rotational axis of the Sun) and the regeneration of a poloidal field from a toroidal one. If there exists a poloidal field, then a toroidal field is generated very efficiently by differential rotation. But the regeneration of the poloidal field poses a problem. For this reason the theory of the turbulent dynamo has been developed (see Moffatt, 1978; Parker, 1979; Krause and Rädler, 1980; Zeldovich et al., 1983; Roberts and Soward, 1992). The central mechanism in this theory is the generation of a mean, or large-scale, electromotive force $\mathcal{E}$ by turbulently fluctuating, or small-scale, parts of velocity and magnetic field, and it is has been found that the presence of kinetic and magnetic helicities is favourable for a so-called

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α-effect, i.e., a non-vanishing component $\mathcal{E}_\alpha = \alpha\langle\mathbf{B}\rangle$ of $\mathcal{E}$ proportional to the mean magnetic field $\langle\mathbf{B}\rangle$; the factor α is in general a tensorial quantity. The densities per unit volume of kinetic, magnetic and current helicity are defined by

$$H_K = \mathbf{v}\cdot(\nabla\times\mathbf{v}),\ H_M = \mathbf{A}\cdot\mathbf{B},\ H_C = \mathbf{B}\cdot(\nabla\times\mathbf{B}), \tag{1}$$

where $\mathbf{v}$, $\mathbf{B}$ and $\mathbf{A}$ denote fluid velocity, magnetic field and a magnetic vector potential. H_M and H_C are closely related.

The α-effect was introduced by Steenbeck et al. (1966) as a mechanism linked with the mean *kinetic* helicity density of turbulent fluid motions, and for isotropic situations, where α is a scalar, traditionally the estimate

$$\alpha \approx -\frac{\tau_{corr}}{3}\langle\mathbf{v}'\cdot(\nabla\times\mathbf{v}')\rangle \tag{2}$$

is quoted, where τ_{corr} is the correlation time of the velocity fluctuations $\mathbf{v}'$ (angular brackets denote averages and primes the corresponding residuals). This estimate is derived under the following approximations and assumptions:

1) The first-order smoothing approximation (FOSA), also known as second-order correlation approximation (SOCA), which consists of neglecting a term $\nabla\times(\mathbf{v}'\times\mathbf{B}' - \langle\mathbf{v}'\times\mathbf{B}'\rangle)$ in the equation for the time evolution of the magnetic fluctuations and thus corresponds to a quasi-linear treatment of the fluctuations.
2) $\langle\mathbf{v}\rangle = \mathbf{0}$.
3) Statistically stationary and homogeneous fluctuations.
4) Magnetic diffusivity $\eta \to 0$.

More recently it was found (Keinigs, 1983; Matthaeus et al., 1986; Rädler and Seehafer, 1990; Seehafer, 1994, 1996) that the α-effect is more directly related to *current* helicity than to kinetic helicity, namely by the relation

$$\sum_{i,j}\alpha_{ij}\langle B_i\rangle\langle B_j\rangle = -\eta\langle\mathbf{B}'\cdot(\nabla\times\mathbf{B}')\rangle\,. \tag{3}$$

For deriving this relation, of the above four conditions only the third one is needed. In the isotropic case, α is a scalar given by

$$\alpha = -\frac{\eta}{\langle\mathbf{B}\rangle^2}\langle\mathbf{B}'\cdot(\nabla\times\mathbf{B}')\rangle. \tag{4}$$

The mean value of the magnetic helicity can be written as the sum of two contributions resulting from the mean and fluctuating magnetic fields, respectively, namely

$$\langle H_M\rangle = H_M^{MEAN} + H_M^{FLUC}, \tag{5}$$

with

$$H_M^{MEAN} = \langle\mathbf{A}\rangle\cdot\langle\mathbf{B}\rangle,\quad H_M^{FLUC} = \langle\mathbf{A}'\cdot\mathbf{B}'\rangle. \tag{6}$$

For the time evolutions of H_M^{MEAN} and H_M^{FLUC} one finds (Seehafer, 1996)

$$\frac{\partial H_M^{MEAN}}{\partial t} = -2\eta\nabla\times\langle\mathbf{B}\rangle\cdot\langle\mathbf{B}\rangle + 2\mathcal{E}\cdot\langle\mathbf{B}\rangle + \left(\frac{\partial H_M^{MEAN}}{\partial t}\right)_{\text{transport}} \tag{7}$$

and

$$\frac{\partial H_M^{FLUC}}{\partial t} = -2\eta\langle\nabla\times\mathbf{B}'\cdot\mathbf{B}'\rangle - 2\mathcal{E}\cdot\langle\mathbf{B}\rangle + \left(\frac{\partial H_M^{FLUC}}{\partial t}\right)_{\text{transport}}\,. \tag{8}$$

These two equations show that the α-effect (appearing through the terms $\mp2\mathcal{E}\cdot\langle\mathbf{B}\rangle$ on the right-hand sides) corresponds to the simultaneous generation of magnetic helicities in the mean field and in the fluctuations, the generation rates being equal in magnitude and opposite in sign. The mean total magnetic helicity, which is an invariant of ideal magnetohydrodynamics, is not influenced by the α-effect. This may equally be considered as a transfer of magnetic helicity between the fluctuating (or small-scale) and the mean (or large-scale) fields mediated by the α-effect, or as a helicity cascade (Frisch et al., 1975; Pouquet et al., 1976; Stribling and Matthaeus, 1990, 1991). The two magnetic helicitities generated by the α-effect, that in the mean field and that in the fluctuations, have either to be dissipated in situ or to be transported out of the dynamo region. The latter may lead to the appearance of magnetic helicity in the solar atmosphere and in interplanetary space. From Eq. (7) it is seen that the magnetic helicity that is accumulated in the mean magnetic field has the sign of $\mathcal{E} \cdot \langle \mathbf{B} \rangle$, i.e., the sign of the scalar α in the isotropic case.

Consider for a moment a situation in which the magnetic fluctuations are statistically homogeneous in space and time. Then in Eq. (8) the term on the left-hand side and the last term on the right-hand side vanish, implying that α is connected to the mean current helicity density of the fluctuations by Eq. (3) in the general case and by Eq. (4) in the isotropic case.

Besides the α-effect, also the Ω-effect (that is, differential rotation) may generate magnetic helicity. In the present paper we concentrate on the contribution of the α-effect and refer to Berger and Ruzmaikin (2000) for a study of magnetic helicity production by the action of differential rotation on magnetic fields in the convection zone; in other studies the generation of magnetic helicity by the differential rotation of the photospheric footpoints of coronal magnetic structures was considered (van Ballegooijen and Martens, 1990; van Ballegooijen et al., 1998; DeVore, 2000; Démoulin et al., 2002).

In contrast to the kinetic helicity in the dynamo region (the convection zone), the current helicity of the dynamo-generated magnetic fields can be observed, namely in the atmosphere (Seehafer, 1990; Rust and Kumar, 1994; Pevtsov et al., 1995; Abramenko et al., 1996; Bao and Zhang, 1998; Zhang and Bao, 1998; Longcope et al., 1998; López Fuentes et al., 2003) and in interplanetary plasma clouds ejected from the Sun (Rust, 1994; Bothmer and Schwenn, 1998). Since the magnetic field is the physical key parameter of the solar atmosphere, the presence of magnetic or current helicity becomes apparent even in purely morphological observations of many of the atmospheric phenomena, for instance as chirality patterns in and around sunspots (Hale, 1927; Richardson, 1941; Ding et al., 1987) and filaments (Martin et al., 1994) or as sigmoid (i.e. S or reverse S shaped) structure of transient brightenings (Rust and Kumar, 1996); for reviews see Zirker et al. (1997), Martin (1998), Rust (2001) and Low (2001). In quantitative analyses, in general use is made of the fact that the magnetic field at superphotospheric levels is approximately force-free, satisfying

$$\nabla \times \mathbf{B} = \alpha_{ff}\mathbf{B}, \quad \alpha_{ff} = \frac{H_C}{\mathbf{B}^2}, \tag{9}$$

where α_{ff} is a scalar that is constant along magnetic field lines but may vary in directions perpendicular to the field lines. The observations indicate that α_{ff} and H_C are predominantly negative in the northern and positive in the southern hemisphere of the Sun. Under the assumption that the magnetic field evolves quasi-statically through successive force-free states with a spatially constant α_{ff}, one has $H_M \cdot H_C > 0$ (see Seehafer, 1990). This is still approximately valid if α_{ff} has a predominant sign in the volume considered, e.g. the atmospheric part of an active region. Thus, we may conclude that the magnetic helicity H_M is also predominantly negative in the northern and positive in the southern hemisphere.

The above discussion of dynamo theory has referred to mean-field dynamo theory only, i.e. to the large-scale dynamo action of small-scale velocity fields. The role of helicity for small-scale dynamos, where the magnetic field and the velocity field vary on comparable scales, is much less clear presently though these dynamos can be studied by direct numerical simulations of the full system of the governing equations (Brandenburg et al., 1996; Brandenburg, 2001). Several strongly helical flows are known to be very dynamo effective on the small scale. The ABC flow $\mathbf{v}_{\mathrm{ABC}}$ (Arnold, 1965; Arnold and Korkina, 1983; Galloway and Frisch, 1986; Seehafer et al., 1996) and the Roberts flow $\mathbf{v}_{\mathrm{R}}$ (Roberts, 1970, 1972; Soward, 1987, 1989, 1994; Rüdiger et al., 1998) are intensively studied examples. These flows can be produced as steady solutions of the incompressible Navier-Stokes equations if an external force field of the ABC type or Roberts type is applied. Feudel et al. (1995) studied the full incompressible magnetohydrodynamic equations with a generalized ABC forcing for which the degree of helicity in the force field (and thus in the generated flow) can be varied by varying a parameter. It was found that, for increasing strength of the forcing, the primary bifurcation from the non-magnetic steady basic flow leads to a dynamo if the degree of helicity in the forcing exceeds a threshold value and to non-magnetic secondary flows for helicities below the threshold value. Thus at least for certain flow families, there is a correlation between helicity and small-scale dynamo action. On the other hand, helicity is certainly not necessary for a small-scale dynamo. So Hughes et al. (1996) found dynamo action for flows with vanishing total kinetic helicity in the volume considered or even identically vanishing helicity density H_K. An important question with respect to solar and planetary dynamos is then how *convective* flows behave in this respect. Demircan and Seehafer (2002) obtained evidence that in the special case of cellular convection in the form of squares helicity is favourable for dynamo action. In the next section of the present paper we report some new results for the small-scale Roberts dynamo, i.e. small-scale dynamo action in driven convection-like rolls, which have also relevance for large-scale dynamo action and helicity generation.

After the treatment of the "laminar" Roberts dynamo we present a straightforward comparison of mean-field model calculations for the α-effect and the current helicity in the solar convection zone with magnetic field observations at atmospheric levels. The natural sources of helicity and the α-effect in cosmical bodies are the action of Coriolis forces on turbulent fluid motions and a stratification of the mean mass density and/or turbulence intensity (Moffatt, 1978; Krause and Rädler, 1980; Rüdiger and Kitchatinov, 1993). The Coriolis forces may be due to rigid or differential rotation. Hitherto in most calculations of the α-effect rigid rotation has been assumed. However, a gradient of the rotation rate, or velocity shear, may significantly influence the turbulence and thus the turbulent electromotive force, besides the role of differential rotation to generate a mean toroidal field from a mean poloidal one. A mean velocity shear may even give rise to an extra contribution to the turbulent electromotive force not vanishing for vanishing mean magnetic field (Yoshizawa, 1990; Yoshizawa and Yokoi, 1993; Blackman and Chou, 1997; Brandenburg and Urpin, 1998; Yokoi, 1999; Blackman, 2000). We here consider the standard α-effect (vanishing for $\langle \mathbf{B} \rangle = \mathbf{0}$) under the influence of differential rotation, using a realistic profile $\Omega(r, \theta)$ of the solar internal rotation rate obtained recently by means of helioseismic inversions by Schou et al. (1998). For further quantities, radial profiles are derived from a standard model of the solar interior (Stix, 2002). We do not include the effect of a mean density stratification in the present study. Yet the turbulence is assumed to be (weakly) compressible. That is to say, density fluctuations and buoyancy effects connected with them are allowed for. Furthermore, the turbulence is assumed to be driven by the Lorentz forces due to prescribed magnetic background fluctuations. These magnetic fluctuations are homogeneous in space and time and their helicity vanishes. Helicity develops in a natural way if the motions driven by them are acted upon by Coriolis forces. We compare the results of the model calculations for the α-effect and the current helicity in the convection zone with magnetic field observations at atmospheric levels in order to adjust unknown or only roughly known model parameters. Specifically, calculated values for the α-effect parameter α and the magnetic field line twist in the convection zone are compared with the force-free coefficient α_{ff} and the current helicity determined from magnetographic measurements in the photosphere. This allows, for instance, an adjustment of the depth in the convection zone at which the α-effect is operating. In a final section we conclude with a brief discussion of the results presented.

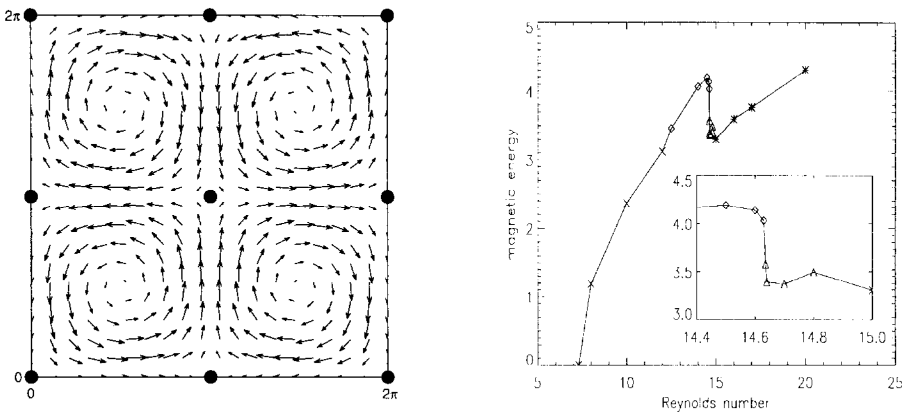
COMPLETE SOLUTION FOR A DYNAMO IN CONVECTION-LIKE ROLLS

In this section we treat a dynamo model based on a flow introduced by G. O. Roberts (1970, 1972). The Roberts flow has recently received renewed interest. On the one hand it resembles the roll solutions of thermal (or solutal) convection. In the convective zones of rotating celestial bodies convection rolls parallel to the axis of rotation tend to be formed (see Busse, 1994). On the other hand the Roberts flow is approximately realized in an ongoing laboratory experiment aimed at demonstrating the dynamo effect under laboratory conditions (Stieglitz and Müller, 1996, 2001). Kinematic studies related to this experiment are due to Apel et al. (1996), who applied mean-field dynamo theory, and Tilgner (1997), who used direct numerical simulation of the induction equation. In both studies the prescribed flow was the Roberts flow and system parameters most suitable for dynamo excitation were determined.

The Roberts flow is given as a family of three-dimensional velocity fields which are independent of one of the spatial coordinates, namely,

$$\mathbf{v}_{\mathrm{R}} = (g \sin x \cos y,\ -g \cos x \sin y,\ 2f \sin x \sin y). \tag{10}$$

f and g are parameters, but we have only used $f = g = 1$ in our calculations. The flow consists of an array of rolls where the fluid spirals up and down in neighboring rolls. To give an impression of the flow structure, a projection of the velocity vectors on the x-y plane is plotted in Figure 1(a). Since the flow is periodic with period 2π in the x and y directions, we consider only the four rolls shown.



(a) (b)

Fig. 1. (a): Projection of the original Roberts flow on the x-y plane. Full black dots indicate stagnation points of the flow. (b): Magnetic energy versus Reynolds number for time-asymptotic states. Steady-state ($\times$), periodic ($\diamond$), torus ($\triangle$), and chaotic solutions ($*$) are marked. The inner small box shows the zoomed region for $R = 14.4 \ldots 15.0$.

We use the incompressible magnetohydrodynamic (MHD) equations in the non-dimensional form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = R^{-1}\nabla^2\mathbf{v} - \nabla p - \frac{1}{2}\nabla\mathbf{B^2} + (\mathbf{B} \cdot \nabla)\mathbf{B} + \mathbf{f}, \tag{11}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = Rm^{-1}\nabla^2\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{v}, \tag{12}$$

$$\nabla \cdot \mathbf{v} = \mathbf{0}, \quad \nabla \cdot \mathbf{B} = \mathbf{0}, \tag{13}$$

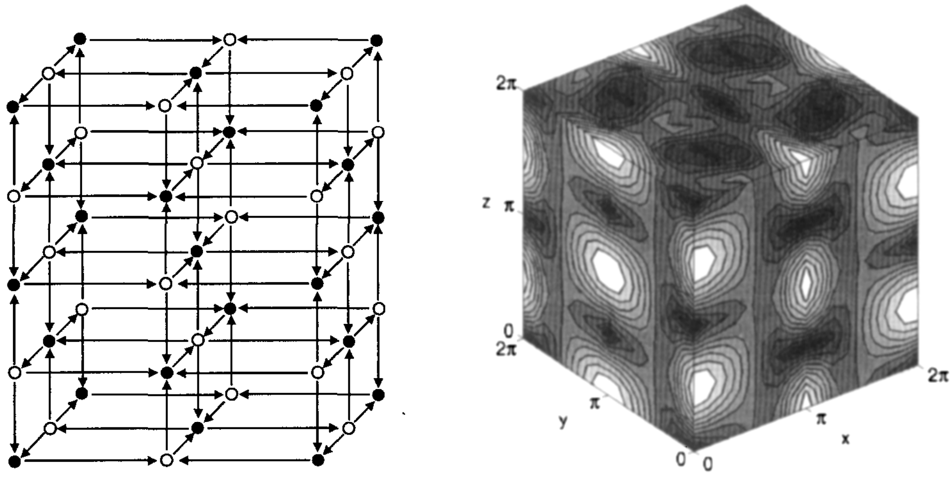
where p is the pressure, R and Rm the kinetic and magnetic Reynolds number, respectively, and $\mathbf{f}$ an external body force. The third and fourth terms on the right-hand side of the Navier-Stokes equation, Eq. (11), constitute the Lorentz force. Equations (13) impose the incompressibility condition on the fluid and ensure the source-free property of the magnetic field. The body force $\mathbf{f}$ on the right-hand side of Eq. (11) has to be specified in the concrete physical context and is the sum of all forces that drive the fluid, as e.g. the buoyancy force in thermal convection, or modify the motion, like the Coriolis force in a rotating star. For simplicity we restrict our investigation to the above system of MHD equations and do not include processes generating the forces. We consider $\mathbf{f}$ as externally applied and given. It pumps energy into the fluid and we look for long lasting magnetic fields, not decaying as a result of the nonlinear coupling of the Navier-Stokes equation and the induction equation, Eq. (12). This phenomenon is called nonlinear dynamo effect.

Applying the external forcing

$$\mathbf{f} = -\nabla^2\mathbf{v}_{\mathrm{R}} = 2\mathbf{v}_{\mathrm{R}} \tag{14}$$

in Eq. (11), the Roberts flow with vanishing magnetic field is a solution of the full MHD equations, Eqs. (11)–(13). It is also the only time-asymptotic steady state for small R; the magnetic Prandtl number $Pm = Rm/R$ is fixed to the value 1 and periodic boundary conditions with period 2π are applied in all three spatial directions.

For increasing Reynolds number a sequence of bifurcations occurs. Figure 1(b) shows the stable solution branches that were obtained by applying continuation techniques and additional simulations. The sudden drop of the magnetic energy in the transition from periodic to quasiperiodic dynamics is conspicuous. For



(a) (b)

Fig. 2. (a): Stagnation points and their connecting heteroclinic orbits after the first bifurcation of the Roberts flow. Full black dots denote α-type stagnation points and empty circles such of type β. (b): Surface-level plot of the modulus of the magnetic field $|\mathbf{B}|$. Bright grey tones indicate regions of strong magnetic field while dark regions correspond to weak fields.

an explanation of this phenomenon we refer to Rüdiger et al. (1998) where also a discussion of the route to chaos may be found.

The key to understanding the primary dynamo bifurcation is given by the stagnation points of the Roberts flow, indicated in Figure 1(a). Due to its independence of the z coordinate, the flow possesses continuous lines of stagnation points which are connected by a family of heteroclinic orbits. The symmetry breaking dynamo bifurcation splits these lines up into a discrete set of sixteen stagnation points. A skeleton of the stagnation points together with the connecting heteroclinic orbits after the bifurcation is sketched in Figure 2(a) (for counting the stagnation points the periodicity has to be taken into account). Following the terminology introduced by Dombre et al. (1986), stagnation points with two negative eigenvalues $(-,-,+)$ are denoted as of α type and such with one negative eigenvalue $(-,+,+)$ as of β type. There are eight stagnation points with two negative eigenvalues and one positive eigenvalue (α type) and eight points with opposite signs of the eigenvalues (β type). There is a close correlation between the locations of these stagnation points and the regions of strong magnetic fields. Figure 2(b) shows a surface-level plot of the modulus of the magnetic field. Bright grey tones indicate regions of strong magnetic fields. Comparing Figures 2(a) and (b), one recognizes that regions of strong fields enclose the stagnation points of β type. Similarly the field is weak in the neighborhood of the stagnation points of α type. A correlation between the stagnation points and the regions of strong magnetic fields has already been found for the ABC dynamo (Galanti et al., 1992; Feudel et al., 1996). However, for the ABC dynamo the strong magnetic fields are concentrated around the α-type stagnation points. The reason for this contrasting behavior of the two dynamos lies in their saturation mechanisms and is discussed by Feudel et al. (2003), who also study the role of the symmetries of the Roberts flow and of layers with chaotic streamlines that are formed between the counter-rotating rolls as a result of the back-reaction of the generated magnetic field on the flow.

An important property of the dynamo is revealed by calculating the horizontally averaged magnetic field

$$\mathbf{B}_h(z) = \int_{\text{periodicity box}} \mathbf{B}\, dx dy\,. \tag{15}$$

Interestingly, one finds $\mathbf{B}_h \neq 0$. Now $\mathbf{B}_h$ is a large-scale field, that is, the non-vanishing of $\mathbf{B}_h$ indicates that the small-scale Roberts dynamo is also a large-scale or mean-field dynamo. After the primary dynamo bifurcation just one Fourier component of $\mathbf{B}_h$, namely that with the vertical wave number $k_z = \mp 1$, is non-vanishing; $\mathbf{B}_h$ and the excited wave vector are:

$$\mathbf{B}_h = (\cos z,\ \sin z,\ 0)\,, \quad \mathbf{k} = (0,\ 0,\ \mp 1) \tag{16}$$

The mean magnetic field is purely horizontal and rotates in a spiral-staircase like fashion. This rotation of the horizontal average of the magnetic field is a well-known property of the *kinematic* Roberts dynamo (Roberts, 1972; Soward, 1987, 1989). Here we find it for the nonlinearly saturated, time-asymptotic magnetic field too.

Let us consider the difference $\mathbf{B} - \mathbf{B}_h$ as the fluctuating part $\mathbf{B}'$ of the magnetic field in the sense of a mean-field theory. Clearly $\mathbf{B}'$ must be strong since otherwise the field concentration at the stagnation points of type β, seen in Figure 2(b), would not come about. Let, further, the Roberts dynamo be considered as an idealized model of the solar dynamo. The field concentrations at the stagnation points of type β can then be interpreted as active regions and the question arises whether $\mathbf{B}_h$ or $\mathbf{B}'$ contain magnetic helicity. The numerical result here is: $H_M(\mathbf{B}_h) < 0$ and $H_M(\mathbf{B}') > 0$; as well as $H_C(\mathbf{B}_h) < 0$ and $H_C(\mathbf{B}') > 0$. That is to say, we find a helicity segregation between the mean field and the fluctuations as predicted by the mean-field theory of the α-effect.

We have attempted here to explain general properties of the nonlinear dynamo, but one has to keep in mind that there is, besides the geometrical idealization, still a Reynolds-number gap of several orders of magnitude between the theoretical model and the dynamo operating in the Sun. Therefore the statistical mean-field approach remains indispensible.

A MEAN-FIELD MODEL WITH APPLICATION TO HELICITY OBSERVATIONS

In this section we report results of a study by Kuzanyan et al. (2003) comparing model calculations for the α-effect and the current helicity in the solar convection zone with magnetic field observations at atmospheric levels.

Review of Some Recent Helicity Observations

Recently in a number of papers systematic studies of magnetographic observations in solar active regions were reported aimed at identifying observational tracers of the α-effect. We here briefly describe the results of Kuzanyan et al. (2000) and Zhang et al. (2002).

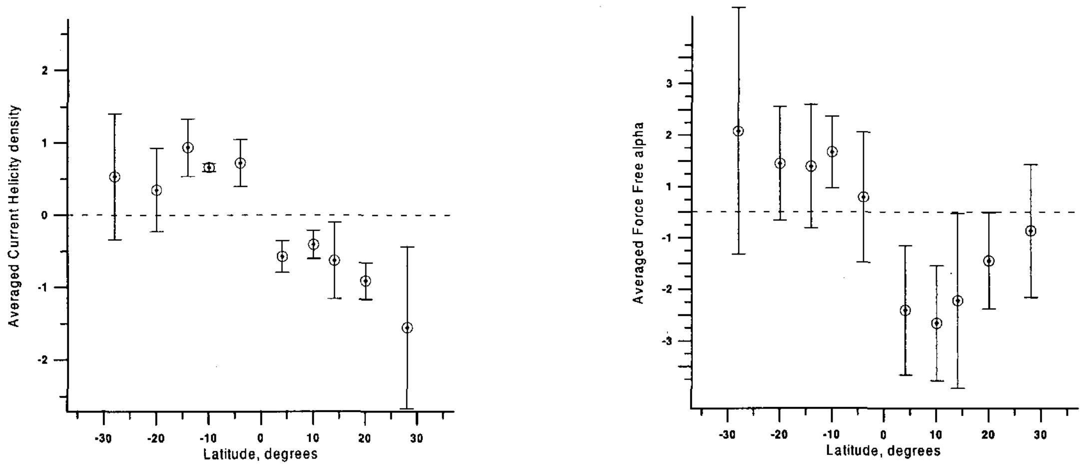
For determining relevant statistical properties of the magnetic field, these authors calculated the two helicity parameters of active regions, namely a mean force-free coefficient $\langle \alpha_{ff} \rangle$ and a mean current helicity density $\langle H_C \rangle \approx \langle \mathbf{B}_{||} \cdot (\nabla \times \mathbf{B})_{||} \rangle$ (the index $_{||}$ denotes longitudinal components of vectors). The caclulations were done neglecting the contribution of the transverse component of the electric current, which cannot be determined from magnetographic observations at a single photospheric level. Mostly, the mean values were obtained by averaging over spatial scales of the order of 5° solar latitude, i.e. a scale slightly smaller than the size of active regions, and times of the order of one Carrington rotation, i.e. 27 days.

The data used were photospheric vector magnetograms of solar active regions observed with the Vector Magnetograph at Huairou Solar Observing Station. The dataset covers the period 1988–1997 and includes most of the large active regions in these ten years. For 422 active regions (one magnetogram per region) both $\langle \alpha_{ff} \rangle$ and $\langle H_C \rangle$ were computed as described, e.g., in Bao and Zhang (1998).

Figures 3(a,b) show that the moduli of the two tracers of the α-effect increase from zero at the equator towards higher latitudes and attain a certain saturation level at middle latitudes. The latitude λ_s at which the saturation occurs lies approximately at $20° - 30°$. We shall compare this observational finding with predictions from model calculations and use it to adjust model parameters.

Theoretical Calculation of $\alpha^{\phi\phi}$ and H_C^{FLUC}

For the solar convection zone, we adopt here a model of a weakly compressible magnetically driven turbulence that is subjected to differential rotation. Though the α-effect parameter α is in general a tensorial quantity, we concentrate on just one of its components, namely the component $\alpha^{\phi\phi}$, which is responsible for



(a) (b)

Fig. 3. Values of the mean current helicity $\langle H_C \rangle = \langle \mathbf{B}_{\|} \cdot (\nabla \times \mathbf{B})_{\|} \rangle$ (a) and the mean twist parameters $\langle \alpha_{ff} \rangle$ (b). Averages were taken over five latitudinal intervals between latitudes 0°, 8°, 12°, 16°, 24°, and 32° in each hemisphere. Vertical lines show 95% confidence intervals.

the generation of the mean poloidal from the mean toroidal field in $\alpha\Omega$-dynamos. Also, assuming that the toroidal component of the mean magnetic field is large compared to the other components, Eq. (3) implies that $\alpha^{\phi\phi}\langle \mathbf{B} \rangle^2 \approx \langle \mathbf{B}' \cdot \nabla \times \mathbf{B}' \rangle = H_C^{FLUC}$ if the fluctuations are homogeneous in space and time (see also Rädler and Seehafer, 1990).

We use first-order smoothing (FOSA) and adopt a polytropic equation of state for the turbulence. In the inertial frame, the (dimensional) equations for the fluctuations then read as follows:

$$\langle \rho \rangle \left[\frac{\partial \mathbf{v}'}{\partial t} + (\langle \mathbf{v} \rangle \cdot \nabla)\mathbf{v}' + (\mathbf{v}' \cdot \nabla)\langle \mathbf{v} \rangle \right] = -\nabla p' + \rho' \mathbf{g} + \langle \rho \rangle \nu \nabla^2 \mathbf{v}' + \frac{1}{\mu_0}(\nabla \times \mathbf{B}_0') \times \langle \mathbf{B} \rangle \tag{17}$$

$$\frac{\partial \rho'}{\partial t} + \langle \rho \rangle \nabla \cdot \mathbf{v}' = 0 \tag{18}$$

$$p' = C_{ac}^2 \rho' \tag{19}$$

$$\frac{\partial \mathbf{B}_1'}{\partial t} = \eta \nabla^2 \mathbf{B}_1' + \nabla \times (\mathbf{v}' \times \langle \mathbf{B} \rangle + \langle \mathbf{v} \rangle \times \mathbf{B}') \tag{20}$$

Here ρ is the mass density, $\mathbf{g}$ the gravitational acceleration and C_{ac} an average value for the speed of sound. The kinematic viscosity ν and the magnetic diffusity η are assumed to stand for their turbulent values rather than for the microscopic ones. The mean motion $\langle \mathbf{v} \rangle$ represents the differential rotation, considered as given, and $\mathbf{B}_0'$ denotes prescribed magnetic background fluctions that are homogeneous in space and time. In the sense of a mixing-length approximation, their spectrum function is chosen as $\mathcal{B}(k,\omega) \sim \langle {\mathbf{B}_0'}^2 \rangle \delta(k - l_{corr}^{-1})\delta(\omega)$ where l_{corr} is the correlation length of the fluctuations or mixing length and the associated correlation time defined by $\tau_{corr} = l_{corr}^2/\nu$ (Kitchatinov, 1991); we also use equal values for the two turbulent diffusivities (i.e., $\nu = \eta$). The background fluctuations satisfy $\langle \mathbf{B}_0' \cdot \nabla \times \mathbf{B}_0' \rangle = 0$, that is, they are non-helical. The last term on the right-hand side of Eq. (17) corresponds to a Lorentz force that drives the turbulent velocity field $\mathbf{v}'$, which in turn, through its action on $\langle \mathbf{B} \rangle$ described by Eq. (20), generates a fluctuating magnetic field $\mathbf{B}'_1$. The total magnetic field is then given by

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{B}'_0 + \mathbf{B}'_1 \,. \tag{21}$$

Using the above equations and a perturbation procedure for solving the continuity equation, Eq. (18), Pipin (2003) derived expressions for $\alpha^{\phi\phi}$ and H_C^{FLUC} in which the energy density of the magnetic background fluctuations appears. If this energy density is replaced by the energy density of the turbulent fluid motions using the equipartition assumption

$$\frac{\left\langle {\mathbf{B}'_0}^2 \right\rangle}{2\mu_0} \approx \frac{\langle \rho \rangle \left\langle \mathbf{v}'^2 \right\rangle}{2} \,, \tag{22}$$

the expressions for $\alpha^{\phi\phi}$ and H_C^{FLUC} become:

$$\alpha^{\phi\phi} = \left[-\cos\theta f_1 \Omega^* + \frac{\cos\theta \sin^2\theta}{2} \frac{\partial \log \Omega}{\partial \log r} f_{\alpha 1} + \frac{\sin\theta}{2} \frac{\partial \log \Omega}{\partial \theta} \left(f_{\alpha 2} - f_{\alpha 1} \sin^2\theta \right) \right] \frac{g \tau_{corr} u_c^2}{C_{ac}^2} \tag{23}$$

$$H_C^{FLUC} = \left[-\cos\theta \psi_0 \Omega^* + \frac{\cos\theta \sin^2\theta}{2} \frac{\partial \log \Omega}{\partial \log r} \psi_{h1} + \frac{\sin\theta}{2} \frac{\partial \log \Omega}{\partial \theta} \left(\psi_{h2} - \sin^2\theta \psi_{h1} \right) \right] \frac{g \langle B_\phi \rangle^2}{C_{ac}^2} \tag{24}$$

Here $u_c = \sqrt{\left\langle \mathbf{v}'^2 \right\rangle}$ is the r.m.s. of the turbulent convective velocity and we have also used $l_{corr}/\tau_{corr} \approx u_c$. Ω^* denotes the Coriolis number, defined by $\Omega^* = 2\Omega_0 \tau_{corr}$, where $\Omega_0 = 2.87 \cdot 10^{-6} s^{-1}$ (surface rotation rate), and the functions f_1, $f_{\alpha 1}$, $f_{\alpha 2}$, ψ_0, ψ_{h1}, ψ_{h2} are given in the Appendix.

Estimates for $\alpha^{\phi\phi}$ and H_C^{FLUC} in the Solar Convection Zone and Comparison with Observations

To get estimates for $\alpha^{\phi\phi}$ and H_C^{FLUC} in the convection zone we use a realistic profile $\Omega(r, \theta)$ of the solar internal rotation obtained recently by means of helioseismic inversions by Schou et al. (1998) in the form of an analytical fit given by Belvedere et al. (2000). Also used are radial profiles of the turbulent convective velocity u_c and of the correlation time τ_{corr} of the turbulence, both derived from a standard model of the solar interior (Stix, 2002). For details of the procedure applied here, which treats the turbulence in a mixing-length approximation with the standard value 1.6 for the mixing-length paramter α_{MLT} (ratio of correlation length to pressure scale height), we refer to Kitchatinov and Rüdiger (1999); see also Küker et al. (1993) and Kitchatinov et al. (2000).

To get a smooth transition from the convection zone to the rigidly rotating radiative interior, the convective velocity in the transition region, i.e. in the thin overshoot layer beneath the bottom of the convection zone, is analytically modeled by

$$v'_o(r) = v'_b \left(1 - \tanh \frac{r_b - r}{d} \right) \tag{25}$$

where r_b marks the bottom of the convection zone, v'_b is the convective velocity at $r = r_b$ and d the half width of the overshoot layer. With this kind of approximation we follow Rüdiger and Brandenburg (1995). We use $r_b = 0.715 R_\odot$ and $d = 0.014 R_\odot$. Inside the overshoot region the Coriolis number Ω^* is fixed to its value at $r = r_b$. The overshooting is followed down to $0.69 R_\odot$. An overview of the profiles of some important quantities in the convection zone, namely the internal differential rotation rate Ω, the Coriolis number Ω^* and the mean convective velocity u_c, is given in Figure 4.

In analogy to the force-free parameter α_{ff} we introduce, for the convection zone, the twist parameter $\alpha_{ff}^{cz} = H_C^{FLUC} / \langle B_\phi \rangle^2$. As a working hypothesis, we assume that the twist α_{ff}^{cz} of the magnetic field in the convection zone propagates upward to atmospheric levels where it is directly observable as the twist α_{ff} of the force-free magnetic field. We wish to emphasize that the physical processes by which magnetic flux generated in the convection zone is transported to the surface are ill-understood at present. Thus the direct comparison of convection-zone and atmospheric twists can only be a trial from which, perhaps, guidance for future studies can be obtained.

From the observational data we could see that the modulus of α_{ff} increases from zero at the equator towards higher latitudes and attains a certain saturation level at middle latitudes. The atmospheric current

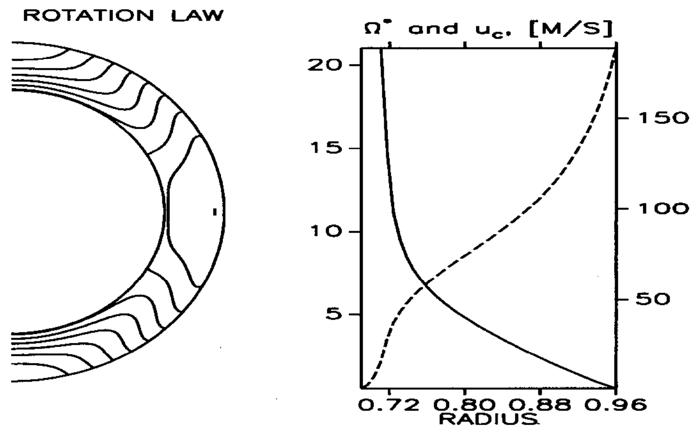


Fig. 4. Left panel: Contours of the rotation rate Ω in the convection zone. Right panel: Radial profiles of the Coriolis number Ω^* (non-dimensional, left scale, solid line) and the convective velocity u_c (in m/s, right scale, dashed line). The radius is given in units of $R_\odot$.

helicity behaves in a similar way. Let us, for each radius in the convection zone, define the latitude $\lambda_s(r)$ at which the twist α_{ff}^{cz} saturates as follows:

$$\lambda_s(r) = \frac{\max\limits_{\theta} \alpha_{ff}^{cz}(r,\theta)}{\left.\frac{\partial \alpha_{ff}^{cz}}{\partial \theta}\right|_{\theta=\frac{\pi}{2}}} \tag{26}$$

Actually $\lambda_s(r)$ as defined by Eq. (26) gives, for fixed r, approximately the inflection point of the curve $\alpha_{ff}^{cz}(\lambda)$, where $\lambda = \pi/2 - \theta$, namely that value of λ at which α_{ff}^{cz} would reach its maximum value if the increase from zero at the equator to the maximum value were linear, with the rate of increase given by the denominator of the fraction on the right-hand side of Eq. (26), i.e. by the increase rate at the equator. We use this seemingly complicated definition of λ_s instead of, say, simply taking the latitude where the maximum (or minimum, respectively) value of α_{ff}^{cz} is attained because at higher latitudes beyond the plateau (not included in the present study) both the current helicity and the force-free parameter could show a complicated behaviour (Pevtsov and Latushko, 2000). The dependence of λ_s on radius is shown in Figure 5(a).

Now let us use the assumption that the α-effect is operating in a very thin spherical shell, i.e. practically at one radius r, and then vary r to get an optimum coincidence between the calculated and the observed saturation latitudes. The best coincidence between $\lambda_s(r)$ and the saturation latitude of the observed atmospheric twist is obviously somewhere between 0.75 and $0.78R_\odot$, that is, rather deep in the convection zone. Figures 5(b,c) show the latitudinal distributions of $\alpha^{\phi\phi}$ and α_{ff}^{cz} at $0.78R_\odot$. They are in good qualitative agreement with the observed latitudinal distributions of the atmospheric current helicity and the twist parameter α_{ff} of the force-free magnetic field; $\alpha^{\phi\phi}$ is opposite in sign to current helicity and twist parameter, in agreement with relations between these quantities found previously (Keinigs, 1983; Seehafer, 1994).

DISCUSSION

The two helicities generated by the α-effect, that in the mean field and that in the fluctuations, have either to be dissipated in the generation region or to be transported out of this region. The latter presumably leads to the observed appearance of magnetic helicity in the atmosphere, and through solar eruptions even in interplanetary space. There has been accumulated strong evidence that the atmospheric and interplanetary magnetic helicity is predominantly negative in the northern and positive in the southern hemisphere. In the mean-field concept of the solar dynamo the mean magnetic field does not reflect the magnetic fields of

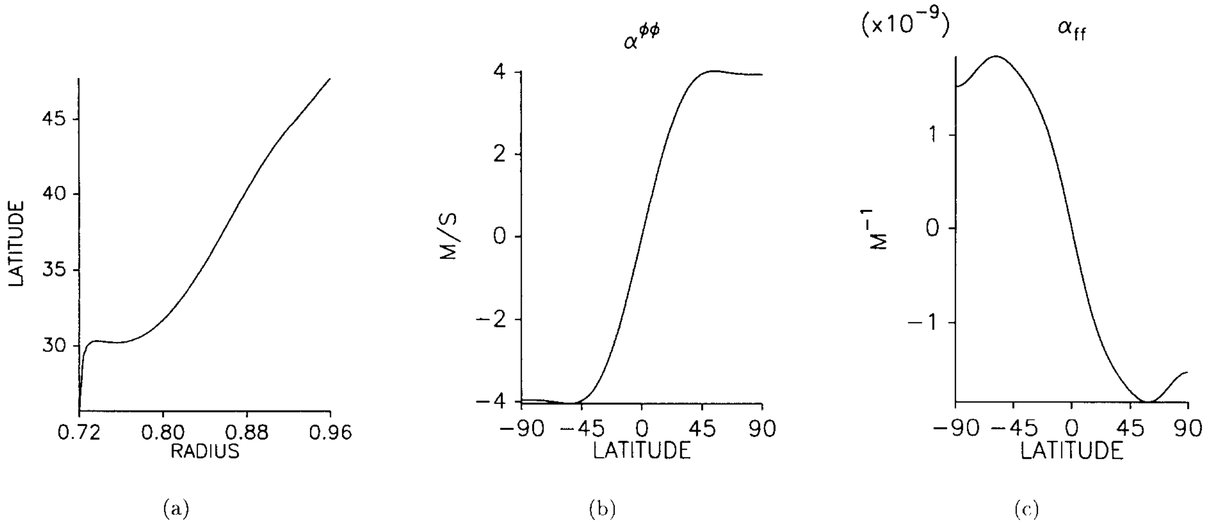


Fig. 5. (a): The dependence of the saturation latitude λ_s of α_{ff}^{cz} (in degrees) on the height in the convection zone (in units of $R_\odot$). (b) and (c): Calculated latitudinal distributions of $\alpha^{\phi\phi}$ (b) and α_{ff}^{cz} (c) at a level of $0.78R_\odot$. $\alpha^{\phi\phi}$ is measured in m/s and α_{ff}^{cz} in m^{-1}.

individual active regions. Although these fields contribute to the mean field, they are presumably mainly fluctuations. The observed magnetic helicities and, for instance, their sign rules thus primarily give information on the fluctuating part of the magnetic field.

In the present review we have included details of two rather different numerical dynamo studies, one for a "laminar" dynamo, the other for a mean-field dynamo. In the "laminar" case the full nonlinear system of the incompressible MHD equations was studied, with an external forcing of the Roberts type driving a flow in the form of an array of convection-like rolls. Though compressibility is certainly important for the solar convection zone, it is generally accepted that the incompressible MHD equations contain all basic ingredients of a dynamo. The equations were solved without further approximations (assuming magnetic Prandtl number $Pm = 1$), resulting in a bifurcation diagram for the time-asymptotic states. The primary bifurcation leads to a dynamo with magnetic fields concentrated at stagnation points of the flow (the β-type stagnation points). We have suggested to interpret the field concentrations as fluctuations in the sense of a mean-field theory — or, with application to the Sun, as active regions. Mean fields are defined by horizontal averages (the direction of the convection-like rolls is the vertical). It is found that there is a segregation of magnetic helicity between the mean field and the fluctuations as predicted by the theory of the α-effect.

Then we have presented mean-field model calculations for the α-effect and the current helicity in the convection zone (with compressibility included) and compared them with magnetic field observations at atmospheric levels. The comparison between model calculations and observations was concentrated on the observational finding that the moduli of the averaged values of the two tracers of the α-effect (α_{ff} and H_C) increase from zero at the equator towards higher latitudes and attain a certain saturation level at middle latitudes (at about $20° - 30°$). The α-effect parameter $\alpha^{\phi\phi}$ and the magnetic field line twist in the convection zone, both calculated using the model, behave in a similar way. On the assumption that the α-effect is operating in a thin spherical shell at a certain depth in the convection zone, this depth was varied to get an optimum coincidence between model and observations, i.e. between calculated and observed saturation latitudes. The coincidence is found to be best for α-effect operation somewhere between 0.75 and $0.78R_\odot$, that is, close to the bottom of the convection zone. The numbers given here must not be taken too literally since neither on the theoretical nor on the observational side saturation latitudes can be determined very accurately.

The models presented here clearly need to be developed in many respects. We only mention that the magnetic Prandtl number $Pm = \nu/\eta$ in the solar convection zone is of the order $10^{-2} \ldots 10^{-6}$ while in the calculations $Pm \approx 1$ is used. In the calculations for the laminar dynamo, furthermore, the Reynolds numbers are much too small, while one has to resort to mixing-length estimates of the turbulent diffusivities in the mean-field treatment where also the use of the turbulent diffusivities instead of the microscopic ones in the governing equations is perhaps too simple. Concerning the observational side, further improved measurements of magnetic helicity would be helpful. One point could be direct or indirect information on transverse photospheric currents in order to get correct values of the current helicity from vector magnetograms. This all must be left for future studies aimed at bridging the still large gap between the theory of the solar dynamo and solar magnetic field measurements.

APPENDIX

The functions f_1, $f_{\alpha 1}$, $f_{\alpha 2}$, ψ_0, ψ_{h1}, ψ_{h2} are defined as follows:

$$
\begin{aligned}
f_1 &= -\frac{1}{4\Omega^{*5}}\left(\Omega^*\left(3+2\,\Omega^{*2}\right)-3\left(1+\Omega^{*2}\right)\arctan(\Omega^*)\right),\\
f_{\alpha 1} &= \frac{1}{160\Omega^{*8}(1+\Omega^{*2})}\Big(10080\Omega^* + 15120\Omega^{*3} + 3431\Omega^{*5} - 2324\Omega^{*7} - 595\Omega^{*9}\\
&\quad +15(-672 - 1232\Omega^{*2} - 505\Omega^{*4} + 137\Omega^{*6} + 85\Omega^{*8} + 3\Omega^{*10})\arctan(\Omega^*)\Big),\\
f_{\alpha 2} &= -\frac{1}{32\,\Omega^{*4}}\left(\Omega^*\left(111+25\,\Omega^{*2}\right)-\left(111+62\,\Omega^{*2}+7\,\Omega^{*4}\right)\arctan(\Omega^*)\right),\\
\psi_0 &= \frac{6\,\Omega^* + 4\,\Omega^{*3} - 6\left(1+\Omega^{*2}\right)\arctan(\Omega^*)}{4\Omega^{*5}},\\
\psi_{h1} &= \frac{1}{96\Omega^{*4}(1+\Omega^{*2})}\Big(\Omega^*(-405 - 228\Omega^{*2} + 73\Omega^{*4}) + 3(135 + 121\Omega^{*2} - 11\Omega^{*4} + 3\Omega^{*6})\arctan(\Omega^*)\Big),\\
\psi_{h2} &= \frac{1}{96\,\Omega^{*4}\left(1+\Omega^{*2}\right)}\left(\Omega^*\left(117+192\,\Omega^{*2}+43\,\Omega^{*4}\right)-3\left(39+77\,\Omega^{*2}+45\,\Omega^{*4}+7\,\Omega^{*6}\right)\arctan(\Omega^*)\right)
\end{aligned}
$$

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REFERENCES

Abramenko, V. I., T. Wang, and V. B. Yurchishin, Analysis of electric current helicity in active regions on the basis of vector magnetograms, *Solar Phys.*, **168**, 75–89, 1996.

Apel, A., E. Apstein, K.-H. Rädler, and M. Rheinhardt, Contributions to the theory of the Karlsruhe dynamo experiment, Research Report, Astrophysikalisches Institut Potsdam, 1996.

Arnold, V. I., Sur la topologie des êcoulements stationnaires des fluides parfaits, *C. R. Acad. Sci. Paris*, **261**, 17–20, 1965.

Arnold, V. I. and E. I. Korkina, The growth of a magnetic field in a three-dimensional steady incompressible flow (in Russian), *Vest. Mosk. Univ. Mat. Mekh.*, **3**, 43–46, 1983.

Bao, S. and H. Zhang, Patterns of current helicity for solar cycle 22, *Astrophys. J.*, **496**, L43–L46, 1998.

Belvedere, G., K. M. Kuzanyan, and D. Sokoloff, A two-dimensional asymptotic solution for a dynamo wave in the light of the solar internal rotation, *Mon. Not. R. Astron. Soc.*, **315**, 778–790, 2000.

Berger, M. A. and A. A. Ruzmaikin, Rate of helicity production by solar rotation, *J. Geophys. Res.*, **10**, 10481–10490, 2000.

Blackman, E. G., Mean magnetic field generation in sheared rotators, *Astrophys. J.*, **529**, 138–145, 2000.

Blackman, E. G. and T. Chou, A vorticity-magnetic field dynamo instability, *Astrophys. J.*, **489**, L95–L98, 1997.

Bothmer, V. and R. Schwenn, The structure and origin of magnetic clouds in the solar wind, *Ann. Geophysicae*, **16**, 1–24, 1998.

Brandenburg, A., The inverse cascade and nonlinear alpha-effect in simulations of isotropic helical hydromagnetic turbulence, *Astrophys. J.*, **550**, 824–840, 2001.

Brandenburg, A., R. L. Jennings, Å. Nordlund, M. Rieutord, R. F. Stein, and I. Tuominen, Magnetic structures in a dynamo simulation, *J. Fluid Mech.*, **306**, 325–352, 1996.

Brandenburg, A. and V. Urpin, Magnetic fields in young galaxies due to the cross-helicity effect, *Astron. Astrophys.*, **332**, L41–L44, 1998.

Busse, F. H., Convection driven zonal flows and vortices in the major planets, *Chaos*, **4**, 123–134, 1994.

Demircan, A. and N. Seehafer, Dynamo in asymmetric square convection, *Geophys. Astrophys. Fluid Dynam.*, **96**, 461–479, 2002.

Démoulin, P., C. H. Mandrini, L. van Driel-Gesztelyi, M. C. López Fuentes, and G. Aulanier, The magnetic helicity injected by shearing motions, *Solar Phys.*, **207**, 87–110, 2002.

DeVore, C. R., Magnetic helicity generation by solar differential rotation, *Astrophys. J.*, **539**, 944–953, 2000.

Ding, Y. J., Q. F. Hong, and H. Z. Wang, A statistical study of the spriral spots on the solar disc, *Solar Phys.*, **107**, 221–231, 1987.

Dombre, T., U. Frisch, J. M. Greene, M. Hénon, A. Mehr, and A. M. Soward, Chaotic streamlines in the ABC flows, *J. Fluid Mech.*, **167**, 353–391, 1986.

Feudel, F., M. Gellert, S. Rüdiger, A. Witt, and N. Seehafer, Dynamo effect in a driven helical flow, *Phys. Rev. E*, 2003, submitted.

Feudel, F., N. Seehafer, B. Galanti, and S. Rüdiger, Symmetry-breaking bifurcations for the magnetohydrodynamic equations with helical forcing, *Phys. Rev. E*, **54**, 2589–2596, 1996.

Feudel, F., N. Seehafer, and O. Schmidtmann, Fluid helicity and dynamo bifurcations, *Phys. Lett. A*, **202**, 73–78, 1995.

Frisch, U., A. Pouquet, J. Léorat, and A. Mazure, Possibility of an inverse cascade of magnetic helicity in magnetohydrodynamic turbulence, *J. Fluid Mech.*, **68**, 769–778, 1975.

Galanti, B., P. L. Sulem, and A. Pouquet, Linear and non-linear dynamos associated with ABC flows, *Geophys. Astrophys. Fluid Dynam.*, **66**, 183–208, 1992.

Galloway, D. and U. Frisch, Dynamo action in a family of flows with chaotic streamlines, *Geophys. Astrophys. Fluid Dynam.*, **36**, 53–83, 1986.

Hale, G. E., The fields of force in the atmosphere of the Sun, *Nature*, **119**, 708–714, 1927.

Hughes, D. W., F. Cattaneo, and E. Kim, Kinetic helicity, magnetic helicity and fast dynamo action, *Phys. Lett. A*, **223**, 167–172, 1996.

Keinigs, R. K., A new interpretation of the alpha effect, *Phys. Fluids*, **26**, 2558–2560, 1983.

Kitchatinov, L. L., Turbulent transport of magnetic fields in a highly conducting rotating fluid and the solar cycle, *Astron. Astrophys.*, **243**, 483–491, 1991.

Kitchatinov, L. L., M. V. Mazur, and M. Jardine, Magnetic field escape from a stellar convection zone and the dynamo-cycle period, *Astron. Astrophys.*, **359**, 531–538, 2000.

Kitchatinov, L. L. and G. Rüdiger, Differential rotation models for late-type dwarfs and giants, *Astron. Astrophys.*, **344**, 911–917, 1999.

Krause, F. and K.-H. Rädler, *Mean-Field Magnetohydrodynamics and Dynamo Theory*, Akademie-Verlag, Berlin, 1980.

Küker, M., G. Rüdiger, and L. L. Kitchatinov, An $\alpha\omega$-model of the solar differential rotation, *Astron. Astrophys.*, **279**, L1–L4, 1993.

Kuzanyan, K., S. Bao, and H. Zhang, Probing signatures of the alpha-effect in the solar convection zone, *Solar Phys.*, **191**, 231–246, 2000.

Kuzanyan, K. M., V. V. Pipin, and N. Seehafer, Alpha effect and the observed twist and current helicity of solar magnetic fields, *Solar Phys.*, 2003, submitted.

Longcope, D. W., G. H. Fisher, and A. A. Pevtsov, Flux-tube twist resulting from helical turbulence: The Σ-effect, *Astrophys. J.*, **507**, 417–432, 1998.

López Fuentes, M. C., P. Démoulin, C. H. Mandrini, A. A. Pevtsov, and L. van Driel-Gesztelyi, Magnetic twist and writhe of active regions, *Astron. Astrophys.*, **397**, 305–318, 2003.

Low, B. C., Coronal mass ejections, magnetic flux ropes, and solar magnetism, *J. Geophys. Res.*, **106**, 25141–25163, 2001.

Martin, S. F., Filament chirality: A link between fine-scale and global patterns, in *New Perspectives on Solar Prominences*, edited by D. Webb, D. Rust, and B. Schmieder, volume 150 of *A. S. P. Conf. Ser.*, pp. 419–429, Astronomical Society of the Pacific, San Francisco, 1998.

Martin, S. F., R. Bilimoria, and P. W. Tracadas, Magnetic field configurations basic to filament channels, in *Solar Surface Magnetism*, edited by R. J. Rutten and C. J. Schrijver, pp. 303–338, Kluwer, Dordrecht, 1994.

Matthaeus, W. H., M. L. Goldstein, and S. R. Lantz, The alpha dynamo parameter and measurability of helicities in magnetohydrodynamic turbulence, *Phys. Fluids*, **29**, 1504–1508, 1986.

Moffatt, H. K., *Magnetic Field Generation in Electrically Conducting Fluids*, Cambridge University Press, Cambridge, England, 1978.

Parker, E. N., *Cosmical Magnetic Fields*, Clarendon Press, Oxford, 1979.

Pevtsov, A. A., R. C. Canfield, and T. R. Metcalf, Latitudinal variation of helicity of photospheric magnetic fields, *Astrophys. J.*, **440**, L109–L112, 1995.

Pevtsov, A. A. and S. M. Latushko, Current helicity of the large-scale photospheric magnetic field, *Astrophys. J.*, **528**, 999–1003, 2000.

Pipin, V. V., The α effect and current helicity for fast sheared rotators, *Geophys. Astrophys. Fluid Dynam.*, **97**, 25–43, 2003.

Pouquet, A., U. Frisch, and J. Léorat, Strong MHD helical turbulence and the nonlinear dynamo effect, *J. Fluid Mech.*, **77**, 321–354, 1976.

Rädler, K.-H. and N. Seehafer, Relations between helicities in mean-field dynamo models, in *Topological Fluid Mechanics*, edited by H. K. Moffatt and A. Tsinober, pp. 157–163, Cambridge Univ. Press, Cambridge, England, 1990.

Richardson, R., The nature of solar hydrogen vortices, *Astrophys. J.*, **93**, 24–28, 1941.

Roberts, G. O., Spatially periodic dynamos, *Phil. Trans. Roy. Soc. London A*, **266**, 535–558, 1970.

Roberts, G. O., Dynamo action of fluid motions with two-dimensional periodicity, *Phil. Trans. Roy. Soc. London A*, **271**, 411–454, 1972.

Roberts, P. H. and A. M. Soward, Dynamo theory, *Ann. Rev. Fluid Mech.*, **24**, 459–512, 1992.

Rüdiger, G. and A. Brandenburg, A solar dynamo in the overshoot layer: cycle period and butterfly diagram, *Astron. Astrophys.*, **296**, 557–566, 1995.

Rüdiger, G. and L. L. Kitchatinov, Alpha-effect and alpha-quenching, *Astron. Astrophys.*, **269**, 581–588, 1993.

Rüdiger, S., F. Feudel, and N. Seehafer, Dynamo bifurcations in an array of driven convectionlike rolls, *Phys. Rev. E*, **57**, 5533–5538, 1998.

Rust, D. M., Spawning and shedding helical magnetic fields in the solar atmosphere, *Geophys. Res. Lett.*, **21**, 241–244, 1994.

Rust, D. M., A new paradigm for solar filament eruptions, *J. Geophys. Res.*, **106**, 25075–25088, 2001.

Rust, D. M. and A. Kumar, Helical magnetic fields and filaments, *Solar Phys.*, **155**, 69–97, 1994.

Rust, D. M. and A. Kumar, Evidence for helically kinked magnetic flux ropes in solar eruptions, *Astrophys. J.*, **464**, L199–L202, 1996.

Schou, J., H. M. Antia, S. Basu, R. S. Bogart, R. I. Bush, S. M. Chitre, J. Christensen-Dalsgaard, M. P. Di Mauro, W. A. Dziembowski, A. Eff-Darwich, D. O. Gough, D. A. Haber, J. T. Hoeksema, R. Howe, S. G. Korzennik, A. G. Kosovichev, R. M. Larsen, F. P. Pijpers, P. H. Scherrer, T. Sekii, T. D. Tarbell, A. M. Title, M. J. Thompson, and J. Toomre, Helioseismic studies of differential rotation in the solar envelope by the Solar Oscillations Investigation using the Michelson Doppler Imager, *Astrophys. J.*, **505**, 390–417, 1998.

Seehafer, N., Electric current helicity in the solar atmosphere, *Solar Phys.*, **125**, 219–232, 1990.

Seehafer, N., Current helicity and the turbulent electromotive force, *Europhys. Lett.*, **27**, 353–357, 1994.

Seehafer, N., Nature of the alpha effect in magnetohydrodynamics, *Phys. Rev. E*, **53**, 1283–1286, 1996.

Seehafer, N., F. Feudel, and O. Schmidtmann, Nonlinear dynamo with ABC forcing, *Astron. Astrophys.*, **314**, 693–699, 1996.

Soward, A. M., Fast dynamo action in a steady flow, *J. Fluid Mech.*, **180**, 267-295, 1987.

Soward, A. M., On dynamo action in a steady flow at large magnetic Reynolds number, *Geophys. Astrophys. Fluid Dynam.*, **49**, 3-22, 1989.

Soward, A. M., Fast dynamos, in *Lectures on Solar and Planetary Dynamos*, edited by M. R. E. Proctor and A. D. Gilbert, pp. 181–217, Cambridge University Press, Cambridge, 1994.

Steenbeck, M., F. Krause, and K.-H. Rädler, Berechnung der mittleren Lorentz-Feldstärke $\overline{\mathbf{v}\times\mathbf{b}}$ für ein elektrisch leitendes Medium in turbulenter, durch Coriolis-Kräfte beeinflußter Bewegung, *Z. Naturforsch.*, **21a**, 369–376, 1966.

Stieglitz, R. and U. Müller, GEODYNAMO — Eine Versuchsanlage zum Nachweis des homogenen Dynamoeffektes, Research Report FZKA-5716, Forschungszentrum Karlsruhe, 1996.

Stieglitz, R. and U. Müller, Experimental demonstration of a homogeneous two-scale dynamo, *Phys. Fluids*, **13**, 561–564, 2001.

Stix, M., *The Sun. An Introduction*, Springer, Berlin, 2002, 2nd edition.

Stribling, T. and W. H. Matthaeus, Statistical properties of ideal three-dimensional magnetohydrodynamics, *Phys. Fluids B*, **2**, 1979–1988, 1990.

Stribling, T. and W. H. Matthaeus, Relaxation processes in a low-order three-dimensional magnetohydrodynamic model, *Phys. Fluids B*, **3**, 1848-1864, 1991.

Tilgner, A., A kinematic dynamo with a small scale velocity field, *Phys. Lett. A*, **226**, 75–79, 1997.

van Ballegooijen, A. A., N. P. Cartledge, and E. R. Priest, Magnetic flux transport and the formation of filament channels on the sun, *Astrophys. J.*, **501**, 866–881, 1998.

van Ballegooijen, A. A. and P. C. H. Martens, Magnetic fields in quiescent prominences, *Astrophys. J.*, **361**, 283–289, 1990.

Yokoi, N., Magnetic-field generation and turbulence suppression due to the cross-helicity effect, *Phys. Fluids*, **11**, 2307–2316, 1999.

Yoshizawa, A., Self-consistent turbulent dynamo modeling of reversed field pinches and planetary magnetic fields, *Phys. Fluids B*, **2**, 1589–1600, 1990.

Yoshizawa, A. and N. Yokoi, Turbulent magnetohydrodynamic dynamo for accretion disks using the cross-helicity effect, *Astrophys. J.*, **407**, 540–548, 1993.

Zeldovich, Y., A. A. Ruzmaikin, and D. D. Sokoloff, *Magnetic Fields in Astrophysics*, Gordon and Breach, New York, 1983.

Zhang, H. and S. Bao, Latitudinal distribution of photospheric current helicity and solar activities, *Astron. Astrophys.*, **339**, 880–886, 1998.

Zhang, H. Q., S. D. Bao, and K. M. Kuzanyan, Twist of magnetic fields in solar active regions, *Astron. Rep.*, **46**, 424–434, 2002.

Zirker, J. B., S. F. Martin, K. Harvey, and V. Gaizauskas, Global magnetic patterns of chirality, *Solar Phys.*, **175**, 27–44, 1997.

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