TOPOLOGICAL STABILITY OF FINITE-LENGTH MAGNETIC FLUX TUBES

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Abstract. It has been suggested that the activity of cosmical magnetic fields is a consequence of a general topological nonequilibrium in the neighbourhood of magnetostatic equilibria. Evidence for this suggestion can be obtained from the Kolmogorov–Arnold–Moser theorem of classical mechanics, applied to the magnetic field line flow as a Hamiltonian system. A finite-length magnetic flux tube, however, always possesses two independent sets of flux surfaces – or, equivalently, the corresponding Hamiltonian system two independent first integrals – and is topologically stable if in the volume occupied by the tube there are no singular (null) points of the magnetic field and the normal field component does not change its sign on the end faces of the tube. Therefore, the concept of nonequilibrium due to flux surface destruction is not applicable to solar atmospheric loops with each end situated in the interior of one polarity of the photospheric normal field component. Further, it seems unlikely that the tearing-mode mechanism can play a role in such loops.

1. Introduction

It has been suggested (Parker, 1972, 1979; Yu, 1973; Tsinganos *et al.*, 1984) that the generally observed (or inferred, respectively) activity of cosmical magnetic fields, with solar coronal heating and (sub-)flares as particular examples, is a consequence of a general topological nonequilibrium in the neighbourhood of magnetostatic equilibria. This suggestion is based on the conjecture that the topology of any magnetostatic field can be changed by arbitrarily small perturbations in such a way that magnetostatic equilibrium is no longer possible.

Mathematically, the notions toplogical equivalence and topological stability are defined as follows (cf., e.g., Arnold, 1973; Chow and Hale, 1982, Chapter 2; Kubiček and Marek, 1983, Appendix C): two fields are termed topologically equivalent if there is a one-to-one continuous mapping with continuous inverse of the volume considered onto itself such that the field lines of the one field are transformed into those of the other. A field $\mathbf{B}(\mathbf{r})$ is termed topologically (or structurally) stable if in a (small) neighbourhood (with respect to an appropriately defined norm in the function space of three-dimensional vector fields) of $\mathbf{B}(\mathbf{r})$ all fields are topologically equivalent to $\mathbf{B}(\mathbf{r})$.

In a plasma of infinitely high electrical conductivity, since the magnetic field is frozen into the plasma and the motion of the medium represents a continuous deformation, the magnetic field evolves through topologically equivalent states. The transition to a non-equivalent topology requires resistivity. On the other hand, resistivity is not sufficient for topological changes. If at a given instant of time a magnetic field has a stable topology, then, by definition, during some (maybe small) time interval about this

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instant the topology cannot change, irrespective of whether the electrical conductivity is finite or infinite.

Magnetic field lines may (a) extend to infinity, (b) be closed, (c) end in a singular (neutral) point of the field, (d) lie on a two-dimensional surface, or (e) wander chaotically through a volume (and may have more of these properties).

In magnetohydrostatic equilibrium the magnetic field **B**, the current density **j**, and the pressure p are related by the equation

$$\mathbf{j} \times \mathbf{B} = \nabla p . \tag{1}$$

If p is a sufficiently smooth function of position, which we can assume, and is not constant in any three-dimensional region, then surfaces p = const. are defined. Equation (1) implies that the magnetic field lines lie on these surfaces. The same holds for the **j**-field and magnetic and current surfaces coincide. If such a surface lies in a bounded volume and is closed, and if neither the **B**-field nor the **j**-field has a singular point on it, it must be topologically equivalent to a torus (Kruskal and Kulsrud, 1958).

If the pressure gradient vanishes, in the low-frequency approximation,

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B} , \tag{2}$$

Equation (1) takes the form

$$\nabla \times \mathbf{B} = \alpha(\mathbf{x}) \cdot \mathbf{B} , \tag{3}$$

with α denoting a scalar function of position – the magnetic field is force-free. From Equation (3) and

$$\nabla \cdot \mathbf{B} = 0 , \tag{4}$$

it follows

$$(\nabla \alpha) \cdot \mathbf{B} = 0 . \tag{5}$$

B-lines and **j**-lines (which coincide) lie again on surfaces, namely the surfaces α = const. The arguments of Kruskal and Kulsrud (1958) can be repeated. Surfaces α = const. exist if α is sufficiently smooth and not constant in three-dimensional regions. If they are bounded and closed and **B** does not vanish on them, then they must be toroids. For force-free fields singular points of the **B**-field are also singular points of the **j**-field (Seehafer, 1986).

In the case of force-free fields with spatially constant α (including current-free fields) the existence of magnetic surfaces cannot be inferred in this way. In fact, as demonstrated by Hénon (1966), these do not in general exist.

It is presently not known whether magnetic fields lacking any symmetry can possess a continuous distribution of flux surfaces. If not, then all magnetohydrostatic fields not force-free with constant α must show such a symmetry, which may be weaker and more subtle than those associated with ignorable spatial coordinates (Low, 1985a, b). It should be noted in this context that the existence of magnetic surfaces is not sufficient for equilibrium. Obviously the symmetry, when existing (and required) is preserved under the frozen-in condition, since this preserves magnetic flux surfaces (though equilibrium may be destroyed under this condition).

By using the Kolmogorov–Arnold–Moser (KAM) theorem of classical mechanics (cf. Whiteman, 1977; Berry, 1978; Arnold, 1978, Appendix 8), Tsinganos *et al.* (1984) have presented evidence that any symmetry-breaking perturbation destroys a finite fraction (a set of nonvanishing measure) of the flux surfaces of a symmetric equilibrium field (that the unperturbed field is an equilibrium field is not explicitly used). This leaves still open the possibility that in the gaps between the preserved (only smoothly deformed) tori the magnetic field is force-free with constant α.

The arguments of Tsinganos *et al.* are not fully conclusive insofar as they use a canonical Hamiltonian representation of the field line flow by means of a Hamiltonian function. For the symmetric equilibrium field such a representation can be obtained and, since the system is integrable, can be transformed to action and angle variables. The perturbations allowed by the canonical perturbation theory – and by Tsinganos *et al.* – are such that the action and angle variables of the unperturbed system are canonical also for the perturbed system. In general, however, the canonical coordinates of the integrable system are not canonical for the perturbed system. This difficulty can be overcome by using a more general noncanonical Hamiltonian formulation (Cary and Littlejohn, 1983), which yields similar results about the destruction of flux surfaces.

The above considerations are only partially relevant for the magnetohydrostatic equilibrium (or non-equilibrium, respectively) in volumes with boundaries intersected by magnetic field lines. This is the case for the solar atmosphere, which is connected by field lines with the much denser subatmospheric layers, and applies similarly to the atmospheres of other stars and to planetary magnetospheres. Magnetohydrostatics is applicable when the velocity of the medium is small compared with the Alfvén velocity or, equivalently, when the kinetic energy density is small compared with the magnetic energy density (Roberts, 1967, p. 21). This condition is satisfied in the solar chromosphere and corona (except during explosive events), but not in and beneath the photosphere.

Since the characteristic velocity, the Alfvén velocity, of the medium above the photosphere is much higher than that of the deeper layers (cf. Priest, 1982, p. 83), in the superphotospheric layers deviations from equilibrium in the form of instabilities can develop so rapidly that the deeper layers remain effectively unchanged on the time-scale of these instabilities. On the other hand, the anchoring of field lines in the dense photosphere acts stabilizing with respect to both ideal and resistive mhd instabilities (Raadu, 1972; Hood and Priest, 1979, 1980; Van Hoven, 1981; Gibons and Spicer, 1981; Mok and Van Hoven, 1982; Einaudi and Van Hoven, 1983; Migliuolo and Cargill, 1983; An, 1984). However, further work is needed to properly treat the photospheric boundary conditions (cf. Low, 1985b).

Resistive instabilities, in particular in loop structures, are considered as important for the explosive release of magnetic energy in solar flares (Spicer, 1976, 1977; Van Hoven, 1979, 1981) and may, occuring at a slower rate, play a role also for coronal heating. The essential feature of resistive instabilities is that they change the field line topology (Furth *et al.*, 1963; Bateman, 1978, Chapter 10; White, 1983). The onset of a topology-changing instability, however, requires a topologically unstable magnetic field. Note that resistive instabilities and topological nonequilibrium as considered by Parker are quite distinct. In a resistive instability the field line topology of the perturbed state, though different from that of the unperturbed state, may be compatible with equilibrium.

In Section 2 of the present paper finite-length magnetic flux tubes, representative of coronal loops, are considered. It is shown that they possess flux surfaces and are topologically stable under rather general conditions.

2. Flux Surfaces and Rectification of Finite-Length Field Line Bundles

We assume the magnetic field $\mathbf{B}(\mathbf{x})$ to be continuously differentiable. Parametric representations $\mathbf{x}(\lambda)$ of individual field lines are obtained as solutions of the equation

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\lambda} = \mathbf{B}(\mathbf{x})\,. \tag{6}$$

The entity of the field lines, the field line flow, can be considered as the phase flow of a mechanical system with the parameter λ as time; because of $\operatorname{div}\mathbf{B} = 0$ this is a Hamiltonian system (cf. Filonenko *et al.*, 1967; Spicer, 1976; Whiteman, 1977; Cary, 1982; Cary and Littlejohn, 1983; White, 1983; Tsinganos *et al.*, 1984; Boozer, 1984; Doveil, 1984; Bernardin and Tataronis, 1985; Thyagaraja and Haas, 1985; Turner, 1985; Salat, 1985). The phase space of the system being three-dimensional, the number of the degrees of freedom is 1.5 – by using $\operatorname{div}\mathbf{B} = 0$, Equations (6) can be brought into a canonical Hamiltonian form with one degree of freedom and an explicit dependence on the canonical time coordinate of the Hamiltonian. When the field possesses some symmetry, the Hamiltonian is independent of time (the system is autonomous).

A function $f(\mathbf{x})$ which is constant along the phase trajectories (the field lines) is called a first integral (a constant of the motion). Here it is assumed that $f(\mathbf{x})$ is continuously differentiable and not identically equal to a constant. A system with a least one first integral is called conservative. Magnetic fields with a dense set of flux surfaces correspond to conservative Hamiltonian systems; the flux surfaces are the level surfaces of a first integral. Obviously, with the exception of the constant-α force-free fields, the field line systems of magnetohydrostatic fields are conservative. The same is true for all symmetric fields, since for them the Hamiltonian itself is a first integral.

Global first integrals are rare, since in general the phase trajectories do not lie fully in the level sets of a function. Locally, however, the situation is quite different. In the neighbourhood of any nonsingular point of the field $\mathbf{B}$, an n-dimensional system of the form given by Equation (6) has just $n - 1$ independent first integrals. This is an immediate consequence of the rectification theorem (Arnold, 1973), which states that, in a sufficiently small neighbourhood V of a nonsingular point $\mathbf{x}_0$, a vector field is diffeomorphic to a spatially constant field $\mathbf{e}_3$. This means that there is a differomorphism (a one-to-one differentiable mapping with differentiable inverse) G of V onto some spatial domain W such that the linear mapping G^* (linear for $\mathbf{x} \in V$ fixed), defined by the matrix

$$G^*_{ij} = \frac{\partial (G(\mathbf{x}))_i}{\partial x_j}, \tag{7}$$

transforms $\mathbf{B}(\mathbf{x})$ into $\mathbf{e}_3$,

$$G^*(\mathbf{x})(\mathbf{B}(\mathbf{x})) = \mathbf{e}_3. \tag{8}$$

This is equivalent to stating that (locally) a coordinate system y_1, y_2, y_3 can be found in which Equation (6) takes the form

$$\frac{\mathrm{d}y_1}{\mathrm{d}\lambda} = \frac{\mathrm{d}y_2}{\mathrm{d}\lambda} = 0, \qquad \frac{\mathrm{d}y_3}{\mathrm{d}\lambda} = 1. \tag{9}$$

y_1 and y_2 and all functions of y_1 and y_2 are first integrals.

The diffeomorphism G, which transforms the field lines into straight lines, can be chosen such that $\mathbf{x}_0$ is a fixed point. Thus any two fields for which $\mathbf{x}_0$ is a nonsingular point are topologically equivalent (even diffeomorphic) and, therefore, topologically stable in the neighbourhood of $\mathbf{x}_0$.

We now consider a finite-length field line bundle. Let the field lines of a bundle start from a connected part S_1 of a plane P_1 and end on a connected part S_2 of a plane P_2. Suppose that in the volume V traversed by the bundle $|\mathbf{B}| \geq b > 0$ and that the normal field component B_n satisfies $B_n \geq b_1 > 0$ on S_1 and $B_n \geq b_2 > 0$ on S_2. We choose a coordinate system x_1, x_2, x_3 such that $x_3 =$ const. in P_1 and x_1, x_2 are a coordinate system in P_1, and set $\lambda = 0$ in P_1. Each point in V lies on just one field line. So it is connected with just one point of S_1 and corresponds to a unique value of the parameter λ. Let the coordinates x_1 and x_2 of the point on S_1 with which the point $\mathbf{x}$ is connected by a field line be given by the functions $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$. The mapping G, defined by

$$G(x_1, x_2, x_3) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \lambda(\mathbf{x})), \tag{10}$$

is a rectifying diffeomorphism. That it is one-to-one is obvious. Further, with the coordinate transformation $y_1 = f_1(\mathbf{x})$, $y_2 = f_2(\mathbf{x})$, $y_3 = \lambda(\mathbf{x})$ Equation (9) is satisfied. G^{-1} is differentiable since the solutions of Equation (6) are differentiable with respect to the initial position (at $\lambda = 0$) and, of course, with respect to λ. Then, the differentiability of G follows from the inverse function theorem (this all is analogous to the proof of the local rectification theorem by Arnold, 1973).

$f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ (and all functions of f_1 and f_2) are first integrals; the surfaces $f_1(\mathbf{x}) =$ const. and $f_2(\mathbf{x}) =$ const. correspondingly flux surfaces. Each field line lies on two independent flux surfaces. Any (smooth) curve on S_1 corresponds to a flux surface, since the field lines starting from this curve form a surface. A dense set of non-intersecting curves on S_1 generates a dense set of flux surfaces.

The surface S_1 is invariant with respect to the mapping G. G can easily be modified such that in the volume W on which the volume V is mapped all field lines, which are straight lines, have a prescribed length; when, e.g., on the right-hand side of Equation (10) the third component is divided by the λ-value of the point on S_2 with which the point $\mathbf{x}$ is connected by a field line, the field lines in W have length unity. Thus all field line bundles with one common end surfaces S_1 can be continuously transformed into each other. Any bundle can be mapped first onto a cylinder with cross section S_1 and length unity and then onto any other bundle; the product mapping is also a diffeomorphism and transforms one bundle into the other. In summary: given two fields $\mathbf{B}(\mathbf{r})$ and $\mathbf{B}'(\mathbf{r})$ and two field line bundles defined by these fields which extend from one common end surface S_1 through volumes V and V', respectively, to end surfaces S_2 and S_2', respectively, (S_1, S_2, S_2' bounded, closed, connected, and, for convenience, plane) with $\mathbf{B} \neq 0$ in V and $\mathbf{B}' \neq 0$ in V' and nonvanishing normal component of $\mathbf{B}$ on S_1 and S_2 and of $\mathbf{B}'$ on S_1 and S_2', there is a one-to-one differentiable mapping of V on V' such that a given field line of $\mathbf{B}$ is mapped on the field line of $\mathbf{B}'$ with the same starting point on S_1. This means, by definition, that the two bundles are topologically equivalent.

Let the field line bundles be loops in the solar atmosphere, defined by a fixed photospheric field line starting point area S_1. Now, for fixed initial position $\mathbf{x}(\lambda = 0)$, the solutions $\mathbf{x}(\lambda)$ to Equation (6) depend continuously on the parameter λ and on changes of the right-hand side, $\mathbf{B}(\mathbf{x})$ (provided these changes are sufficiently smooth). If the above suppositions on $\mathbf{B}$ in the bundle and on its end faces are satisfied for a given field, then for sufficiently small perturbations of the field the change of the volume traversed by the field lines (starting from S_1) is so small that they are also satisfied for the perturbed field. Consequently perturbed and unperturbed bundle are topologically equivalent. Thus, by definition, such bundles are topologically stable.

Until now one loop end was kept fixed. However, the above considerations can easily be generalized to include changes of S_1, provided there is a one-to-one continuous mapping between S_1 and the perturbed footpoint area S_1'. Then there is a continuous mapping between the two cylinders with cross sections S_1 and S_1', respectively, into which unperturbed and perturbed loop, respectively, can be transformed. The suppositions needed remain satisfied for small changes of the footpoint area, since the solutions to Equation (6) depend continuously also on the initial position $\mathbf{x}(\lambda = 0)$.

3. Discussion

It has been shown in Section 2 that finite-length magnetic flux tubes possess two independent sets of flux surfaces and are topologically stable if in the volume occupied by the flux tube there are no singular (null) points of the magnetic field and if on the end faces of the tube the normal field component does not change its sign. Therefore, the concept of nonequilibrium due to flux surface destruction is not applicable to solar atmospheric loops with both ends situated in the interior of one polarity of the photospheric normal field component. Such loops evolve through topologically equivalent states irrespective of whether the magnetic field is frozen-in or not.

The result obtained is in particular applicable to the uniform field extending in the

x_3-direction between two parallel plates at $x_3 = \mp L$ considered by Parker (1972, 1979). Of course a perturbed state, in spite of the existence of magnetic flux surfaces, will in general not be a magnetohydrostatic equilibrium state and Parker's conjecture may be valid. However, the term 'topological' nonequilibrium is inadequate for the situation.

Equally inadequate for finite-length flux tubes are the topological notions 'island formation', 'island coalescence', and 'field line stochasticity'. In toroidal geometry, these have the following meaning: given a set of nested toroidal flux surfaces around a central closed field line, the magnetic axis, the destruction of a part of the flux surfaces results in the formation of magnetic islands or stochastic field lines (or both) between preserved flux surfaces. A magnetic island is a new set of nested flux surfaces with its own local magnetic axis. Stochastic field lines do not lie on surfaces. Both phenomena are connected with the presence of infinitely long field lines. They can occur also in infinitely long straight cylinders. In contrast to finite-length flux tubes, these allow non-equivalent field line topologies.

Formation and growth of magnetic islands are characteristic of tearing modes, which are considered as a possible (and likely) mechanism of the conversion of magnetic energy into particle energies in solar flares (Spicer, 1977; Van Hoven, 1981). The islands originate at so-called mode-rational surfaces. In the symmetry of a cylinder, straight or closed to a torus, it is usual to decompose perturbations of the equilibrium quantities into Fourier components $f(r)\exp(im\theta + ikz)$ (m, k in general not integers), where r is the distance from the axis, θ the poloidal angle, and z the distance along the axis, and to evaluate the stability of each excitation separately. The condition for magnetic tearing is the existence of a flux surface (the mode-rational surface) on which

$$\frac{m}{r}B_\theta + kB_z = 0\,, \tag{11}$$

i.e., on which the perturbation (its vector components in the r–θ–z-coordinate system) is constant along the field lines of the equilibrium magnetic field, or the lines of constant perturbation coincide with the field lines, respectively. For the toroidal geometry this implies that on the mode-rational surface the field lines are closed. According to the Kolmogorov–Arnold–Moser theorem just those invariant tori of an integrable Hamiltonian system are, or may be, destroyed by a perturbation on which the phase-space trajectories close upon themselves. A perturbation resonant with such a rational torus destroys not only this torus but all tori in a layer about it, the width of this layer being small when the perturbation is small.

When observing the field lines of a finite-length flux tube which is a part of a global field configuration with nested toroidal flux surfaces, one does not detect the formation of islands or of a stochastic layer of the global configuration. The finite-length bundle evolves through equivalent topologies. Resistive diffusion of the magnetic field, however, which is necessary for the change of the global topology, may be observed also in the finite-length part. Similarly, when the topological change is accompanied by the release of magnetic energy, energy conversion can be observed locally.

Now the boundary conditions for the instability of a coronal loop follow from the field-line tying in the photosphere. If the photosphere is considered as a perfectly conducting rigid plate, then both velocity and magnetic field cannot change at the loop ends. This excludes, in particular, tearing modes, since the resonance condition, Equation (11), then implies the vanishing of a perturbation in the whole loop (cf. Gibons and Spicer, 1981), provided that all field lines are anchored in the photosphere. On a mode-rational surface the field lines must be detached from the photosphere. In cylindrical symmetry, the axial field component must vanish on such a surface, and the unstable modes are those with $m = 0$ in Equation (11) (cf. Mok and Van Hoven, 1982).

Thus, maybe, for the solar loops the concept of 'modes' and associated singular surfaces should be discarded. For the resistive gravitational instability, Roberts and Taylor (1965; cf. also Dagazian, 1976), who did not Fourier analyze in the direction of the main field, have demonstrated the possibility of excitations not localized about a singular surface, so-called quasi-modes. Spicer (1976) has pointed to the possible role of (hypothetical) tearing quasi-modes in solar flares.

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