ON THE MAGNETIC FIELD LINE TOPOLOGY IN SOLAR ACTIVE REGIONS

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Abstract. The field lines of closed magnetic structures above the photosphere define a mapping from the photosphere to itself. This mapping is discontinuous, and the field line connectivity to the boundary can change discontinuously in response to continuous changes of field strength and direction, if field lines either end in a singular point of the field or are tangential to the photosphere at one end. Whereas the general existence of singular points is questionable, the field has typically a cell structure due to the presence of segments of the zero line of the photospheric longitudinal field on which the transversal field is directed from negative (pointing into the Sun) to positive fields. The cell boundaries are made up of field lines which all touch the photosphere on one of these line segments. Within each of the cells the field line mapping is continuous. When during a slow evolution a substantial part of a coronal loop or of an arcade has passed from one cell into another a fast dynamic instability may set in which was previously prevented by the anchoring of field lines in the dense photosphere.

1. Introduction

According to Sweet (1958, 1969; cf. also Syrovatskii, 1981; Bratenahl and Baum, 1976) the magnetic field of a solar active region with two bipolar sunspot groups contains in general two neutral (singular, null) points, where the field vanishes. These are situated in the photosphere, which is assumed to be a plane. The field above the photosphere is divided into four topologically distinct flux systems by two intersecting separating surfaces. These surfaces are made up of field lines which for each surface end or start, respectively, in one of the neutral points; the line of intersection of the two surfaces is a field line joining the two neutral points.

Sweet derived this model from the general statement that photospheric flux concentrations share their flux among several neighbouring flux concentrations of opposite sign. He considered four flux tubes protruding through the photosphere and representing sunspots, assuming the photospheric normal field component to vanish outside the area of the four spots. Then outside the spots the photosphere is a magnetic surface, i.e., the field lines lie in the photosphere. However, that the two-dimensional field on this magnetic surface has two singular points can hardly we derived from $\operatorname{div}\mathbf{B} = 0$ alone.

Let the x_3 direction of a system of Cartesian coordinates x_1, x_2, x_3 be perpendicular to the photosphere and upward in the atmosphere. If on the whole periphery of each of the two positive (with positive field component B_3) spots the photospheric B_1–B_2 field has a positive component along the exterior normal, and is correspondingly directed into the spots on the peripheries of the negative spots, then the arguments of Sweet (1958) apply and in general there exist two singular points in the photosphere. For flux tubes

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perpendicular to the photosphere this seems plausible (cf. Sweet, 1958, Figure 1). If, however, a flux tube emerges obliquely through the photosphere, the photospheric B_1–B_2 field is directed into the spot on a part of the spot boundary and out of the spot on another part. It may occur that a photospheric field line followed along its positive direction leads out of a positive and into a negative spot.

Also the following consideration indicates that Sweet's model is not necessarily typical of the field configuration in solar active regions. Let the field have Sweet's configuration and then shift the photosphere upward by a small amount while leaving the field unchanged. Then the resulting configuration has no neutral points in or above the photosphere, though flux shearing is still present.

For a potential field in and above the photosphere, Molodensky and Syrovatskii (1977) generalized the four-component model of Sweet to an arbitrary number of spots. Using the concept of the Poincaré index, they showed that if spots are assumed to be singular points of node type of the photospheric B_1–B_2 field, in general the difference between the number of spots and the number of saddle (X) points of the two-dimensional photospheric field is two (that is that for more than two spots such saddle points exist). In Section 2 of the present paper it is shown that for deriving this the supposition that the magnetic field is a potential field can be given up. If the photosphere is a magnetic surface outside the spots, the photospheric saddle points are also singular points of the three-dimensional field.

Baum and Bratenahl (1980) calculated numerically the field due to four point charges situated in the photosphere and assumed to represent sunspots. They obtained Sweet's configuration. The field generated by point charges (or dipoles, respectively), often used to model the magnetic field above the photosphere (cf., e.g., Syrovatskii, 1969; Priest and Raadu, 1975; Baum *et al.*, 1979), is discussed in Section 3. If the point charges are located below the photosphere, the field in and above the photosphere contains neutral points only for special arrangements of the charges.

Recently, Seehafer (1985a) presented an example for solar flares possibly caused by a change of the magnetic field line topology. Using a photospheric magnetogram of a solar activity complex as boundary data, force-free magnetic fields with spatially constant α, α defined by the equation $\nabla \times \mathbf{B} = \alpha\mathbf{B}$, were calculated. When the parameter α was continuously varied, it was observed that field lines changed their connectivity to the boundary discontinuously. Let, as in the model of Sweet, the field above the photosphere be partitioned into cells such that within each cell the mapping of the boundary on itself, defined by the field lines, is continuous. During an evolution of the field, the separating surfaces between the cells are deformed and displaced, accompanied by the passage of field lines from one cell into another and a discontinuous change of the connectivity to the boundary of these field lines.

In Section 4, further numerical studies of the example of Seehafer (1985a) are presented in order to elucidate the effect observed. Whereas the cell structure seems to be typical of the magnetic field of active regions, the existence of neutral points remains questionable.

2. Singular Points and Singular Field Lines

Parametric representations $\mathbf{x}(t)$ of individual magnetic field lines are obtained as solutions of the equation

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{B}(\mathbf{x}). \tag{1}$$

Equation (1) can be considered as describing a mechanical system with the parameter t as time and the field lines as phase space trajectories. Because of $\operatorname{div}\mathbf{B} = 0$ this is a Hamiltonian system, i.e., the phase volume is preserved under the field line flow.

We suppose that $\mathbf{B}(\mathbf{x})$ is continuously differentiable. Through any point $\mathbf{x}_0$ there is a unique solution $\mathbf{x}(\mathbf{x}_0, t)$, which, for given finite t, is continuous and continuously differentiable with respect to the initial position $\mathbf{x}_0$ (Arnold, 1973). If the right-hand side of Equation (1) depends continuously on some parameter or vector of parameters $\mathbf{a}$, $\mathbf{B} = \mathbf{B}(\mathbf{x}, \mathbf{a})$, then the solution $\mathbf{x}(t, \mathbf{x}_0, \mathbf{a})$ depends continuously on $\mathbf{a}$ (and is differentiable with respect to $\mathbf{a}$ if $\mathbf{B}$ is correspondingly differentiable). The parameter t for the description of the entity of the field lines, the field line flow, is determined except for an additive constant.

Each singular point $\mathbf{x}_0$ represents itself a field line, $\mathbf{x}(t) \equiv \mathbf{x}_0$, or an equilibrium solution of the mechanical system, respectively (cf. Arnold, 1973; Minorsky, 1962; Bogoliubov and Mitropolsky, 1961). There is a one-to-one correspondence between the singular points of $\mathbf{B}$ and the equilibrium solutions of Equation (1). Because of the uniqueness of the solutions, field lines can approach singular points only as $t \to \infty$ or $t \to -\infty$.

Consider a finite or infinite volume V. A field line intersecting the boundary ∂V of V must return to ∂V unless it is of infinite length in V or ends in a singular point. Consider further a point $\mathbf{x}_0$ on ∂V in which the field component normal to ∂V does not vanish and a (piece of a) curve on ∂V through $\mathbf{x}_0$. If the field line through $\mathbf{x}_0$, when followed along its positive or negative direction into V, ends in a point $\mathbf{x}_1$ on ∂V with nonvanishing normal field component, then in a sufficiently small neighbourhood of $\mathbf{x}_0$ the curve through $\mathbf{x}_0$ is mapped onto a continuous curve through $\mathbf{x}_1$ by the field lines.

Thus, when the mapping of a curve on which the field component normal to ∂V does not vanish is discontinuous at some point on the curve, the field line through this point must either lead to a singular point or to a point on ∂V with a vanishing normal field component. In the first case we shall speak of a singular, in the second of a quasi-singular field line. The effect of quasi-singular field lines is illustrated in Section 4. That the presence of singular points can lead to a discontinuity of the field-line mapping is connected with the fact that the theorem on the continuous dependence of the solutions of Equation (1) on the initial conditions is valid only for finite t, whereas a singular field line, also when of finite length, approaches the singular point as $t \to \infty$ or $t \to -\infty$.

Fully analogous considerations can be made when not the field line starting point on ∂V but some parameter a on which the field continuously depends is varied. Choose a point $\mathbf{x}_0$ on ∂V and an interval of a such that for all values of a the normal field component at $\mathbf{x}_0$ does not vanish. With variation of a the point on which $\mathbf{x}_0$ is mapped describes a curve on ∂V. If this curve is discontinuous for some value a_0 of a, then for $a = a_0$ the field line starting from $\mathbf{x}_0$ must be a singular or a quasi-singular field line.

For a two-dimensional vector field on a plane, the Poincaré index (cf. Arnold, 1973; Bogoliubov and Mitropolsky, 1961) of a closed curve not running through a singular point is defined as the number of rotations of the field vector around the curve, i.e., as the angle in units of 2π by which the field vector changes around the curve. The index of a curve not enclosing a singular point is zero. The index of an isolated singular point is defined as the index of a curve of small diameter enclosing the point. The index of a curve is equal to the sum of the indexes of the singular points enclosed by the curve.

In the general case, for non-singular gradient matrix $\partial B_i/\partial x_j$, singular points of two-dimensional fields have the index $+1$ or -1, -1 only for a saddle (X) point.

Molodensky and Syrovatskii (1977) assume the field in (and above) the photosphere to be a potential field. Then at infinity the field has dipole character (if the total dipole moment does not vanish), and the index of a curve of sufficiently large diameter is two, that is, the sum of the indexes of the singular points of the photospheric B_1–B_2 field is two. If spots are then assumed to be singular points of node type, the difference between the number of spots and the number of photospheric saddle points is two.

On a spherical photosphere the supposition on the index sum of the two-dimensional photospheric field is strictly valid. This follows from the theorem (cf. Arnold, 1973) that the sum of the indexes of the singular points of a vector field on a spherical surface is two, independent of the choice of the vector field. (This independence is connected with the fact that a spherical surface is a compact manifold, and does not apply to a plane, which is not compact.)

3. Fields Due to Point Charges

The field generated by two point charges, or one dipole, located below the photosphere has no singular point in or above the photosphere. The photospheric B_1–B_2 field has two singular points of node type (only one when the dipole moment is perpendicular to the (plane) photosphere).

We shall consider the field due to four coplanar point charges, two positive and two negative, of equal strength. The total dipole moment vanishes if (and only if) the point charges form two anti-parallel dipoles (a quadrupole), i.e., are situated at the corners of a parallelogram with neighbouring charges having opposite sign.

Let the four point charges be located in the photospheric plane. If the total dipole moment does not vanish (cf. Baum *et al.*, 1979, Figure 1), the Poincaré index of a photospheric curve of sufficiently large diameter is two and there are two saddle points in the photosphere, which are also singular points of the three-dimensional field since the photosphere is a magnetic surface. In the case of a quadrupole (cf. Syrovatskii, 1969, Figure 2) the index sum is three and there is one photospheric saddle point. The second saddle point may be imagined as placed at infinity; in the case of a spherical photosphere there exist two saddle points in any case.

Now let the charges be situated at equal depth below the photosphere. If they are close to the photosphere, the photospheric B_1–B_2 field has still the same configuration, i.e., contains four nodes and one or two saddle points. Whether there are three-dimensional singular points in or above the photosphere is, however, less obvious.

For special arrangements of the charges such singular points exist. The field due to four point charges arranged colinearly with alternating sign (Baum *et al.*, 1979) is axially symmetric with a circular line consisting of singular points, which is in part above the photosphere if the charges are sufficiently close to the photosphere (cf. Syrovatskii, 1969, Figure 1). When, as a second example, the charges are situated at the corners of a rectangle and neighbouring charges have opposite sign, there is a straight line perpendicular to the plane of the rectangle on which the field vanishes for symmetry reasons (cf. Syrovatskii, 1969, Figure 2).

The field due to four point charges in the plane $x_3 = 0$, $+e$ at positions $\mathbf{a}$ and $\mathbf{c}$ and $-e$ at positions $\mathbf{b}$ and $\mathbf{d}$, is given by

$$\mathbf{B}(\mathbf{x}) = \frac{e(\mathbf{x}-\mathbf{a})}{R_\mathbf{a}^3} - \frac{e(\mathbf{x}-\mathbf{b})}{R_\mathbf{b}^3} + \frac{e(\mathbf{x}-\mathbf{c})}{R_\mathbf{c}^3} - \frac{e(\mathbf{x}-\mathbf{d})}{R_\mathbf{d}^3}\,, \tag{2}$$

with $R_\mathbf{a} = |\mathbf{x} - \mathbf{a}|$, etc. For $x_3 \neq 0$ the condition $B_3(\mathbf{x}) = 0$ takes the form

$$\frac{1}{R_\mathbf{a}^3} - \frac{1}{R_\mathbf{b}^3} + \frac{1}{R_\mathbf{c}^3} - \frac{1}{R_\mathbf{d}^3} = 0\,. \tag{3}$$

Then the conditions $B_1(\mathbf{x}) = 0$ and $B_2(\mathbf{x}) = 0$ imply

$$\frac{-a_1}{R_\mathbf{a}^3} + \frac{b_1}{R_\mathbf{b}^3} - \frac{c_1}{R_\mathbf{c}^3} + \frac{d_1}{R_\mathbf{d}^3} = 0\,, \tag{4}$$

$$\frac{-a_2}{R_\mathbf{a}^3} + \frac{b_2}{R_\mathbf{b}^3} - \frac{c_2}{R_\mathbf{c}^3} + \frac{d_2}{R_\mathbf{d}^3} = 0\,. \tag{5}$$

Equations (4) and (5) cannot be satisfied if in the plane $x_3 = 0$ there exists a straight line separating the two positive charges from the two negative (if, e.g., this straight line is taken as the x_2-axis, obviously Equation (4) cannot be satisfied). For example the configuration studied numerically by Baum and Bratenahl (1980) belongs to this category and has consequently no singular point above the plane of the charges.

We suppose that a straight line separating positive and negative charges does not exist and restrict the problem by locating the charges at the corners of a parallelogram (then neighbouring charges must have opposite sign). Without loss of generality we can write

$$a_1 = a_2 = b_1 = 0\,, \qquad c_1 = d_1\,, \qquad c_2 - d_2 = b_2\,. \tag{6}$$

Then from Equations (3)–(5) we have

$$R_\mathbf{a} = R_\mathbf{b} = R_\mathbf{c} = R_\mathbf{d}\,. \tag{7}$$

That is, the parallelogram must be a rectangle.

It can be stated that singular points outside the plane of the charges exist only for special arrangements of the charges, though not all possible configurations have been considered.

4. Activity Complex HR 16862, 16863, 16864

Seehafer (1985a) calculated force-free magnetic fields for the activity complex HR 16862, 16863, 16864 at the end of May 1980. In the model used strength and direction of the field depend continuously on the parameter α of the force-free field. It was observed that, for a fixed starting point of a field line on the boundary (photosphere), with variation of α the position of the end point changed discontinuously for some critical value of α. According to the discussion in Section 2, the field line considered must be singular or quasi-singular for the critical parameter value.

Let for fixed starting point $\mathbf{x}_0$ the end point $\mathbf{x}_1$ jump for $\alpha = \alpha_c$,

$$\lim_{\alpha \to \alpha_c + 0} \mathbf{x}_1(\alpha) \neq \lim_{\alpha \to \alpha_c - 0} \mathbf{x}_1(\alpha) .$$

If the field component normal to the boundary is different from zero at $\mathbf{x}_0$, $\mathbf{x}_1(\alpha_c - 0)$, and $\mathbf{x}_1(\alpha_c + 0)$, the field line considered must be a singular field line, i.e., there must exist a singular point. Thus, when systematically applied, one has a method to prove the existence of singular points. If, however, $\mathbf{x}_1(\alpha_c - 0)$ or $\mathbf{x}_1(\alpha_c + 0)$ is situated on a zero line of the normal field component, the existence of a singular point cannot be inferred; whether the limit point on the zero line is perhaps itself a singular point can be easily checked.

For the activity complex HR 16862, 16863, 16864 the behaviour of a number of individual field lines, selected because they changed their connectivity to the boundary with variation of α, has been studied in detail. All these field lines proved to be quasi-singular field lines; evidence for singular points has not been found.

At those points of the zero line of the photospheric normal component where a quasi-singular field line is tangential to the photosphere, the photospheric transversal field is not directed from the area of positive to the area of negative fields but from negative to positive fields. Segments of the zero line consisting of such points are not bridged by field lines above the photosphere; they may be imagined to be bridged below the photosphere.

Figure 1 of the present paper should be looked at in conjunction with Figure 1 of Seehafer (1985a). It shows the zero line of the longitudinal magnetogram of the activity complex, whereas in the previous paper only fields stronger than 20 G are represented. Segments where the photospheric B_1–B_2 field is directed from negative to positive fields are drawn as dashed lines. Each of these segments causes discontinuous field line mapping. The photospheric transversal field on the zero line was calculated from the longitudinal magnetogram using the method of Seehafer (1978) with $\alpha = 0.005$ arc sec^{-1}.

A cross in Figure 1, in the positive polarity of HR 16862, marks the photospheric starting point of a selected field line. When the parameter α is increased from zero while

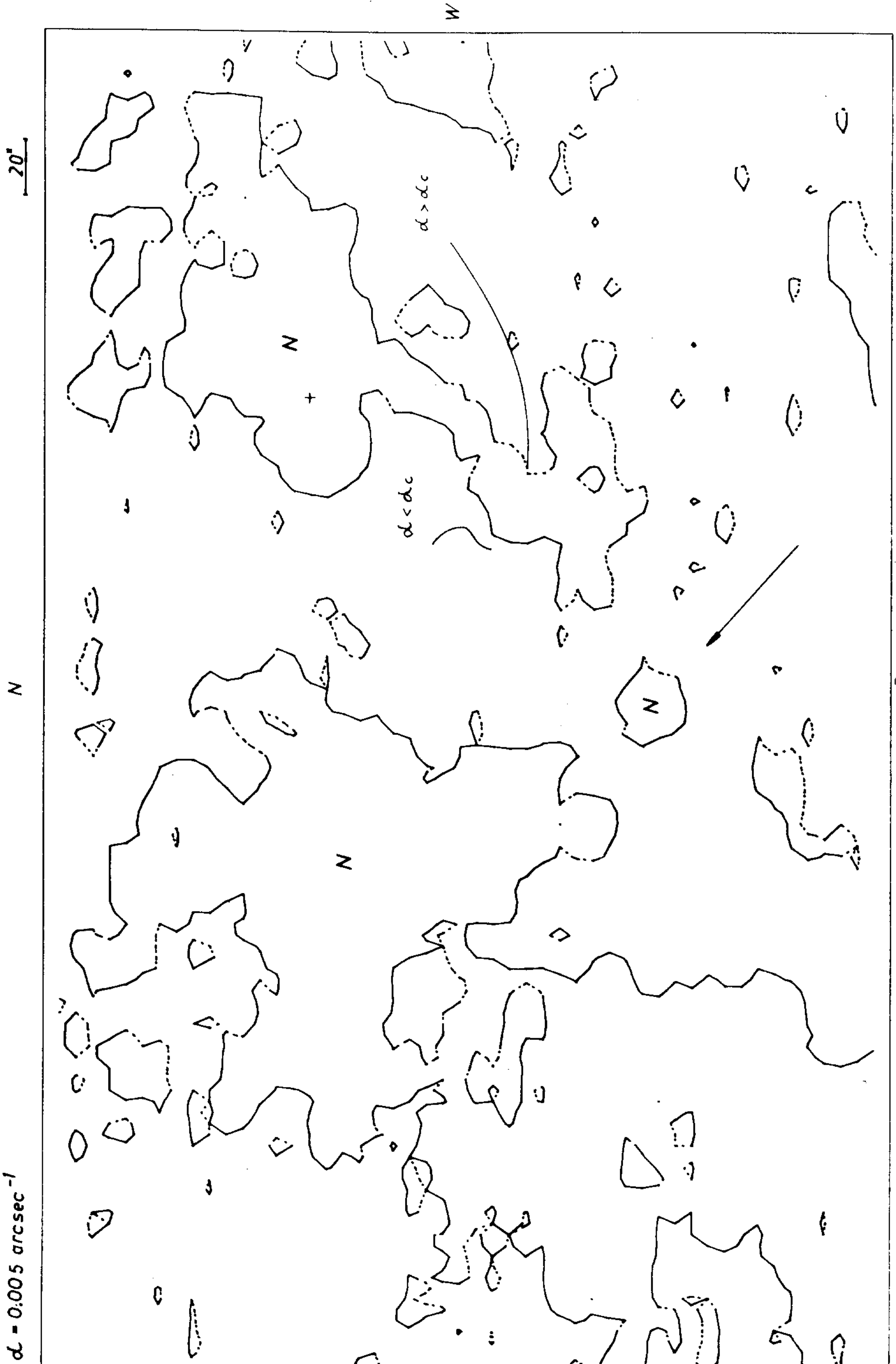


Fig. 1. Zero line of the photospheric longitudinal field (magnetographic measurement) of the activity complex HR 16862, 16863, 16864. On dashed line segments the photospheric transversal field is directed from negative to positive longitudinal fields. The letter N denotes positive (north) polarity. The two open lines are the two parts of the curve described by the end point of the field line starting in the point marked by a cross with variation of α for $\alpha < \alpha_c$ and $\alpha > \alpha_c$, respectively. The arrow indicates a positive polarity satellite spot forming a delta configuration with the negative polarity main spot of HR 16864.

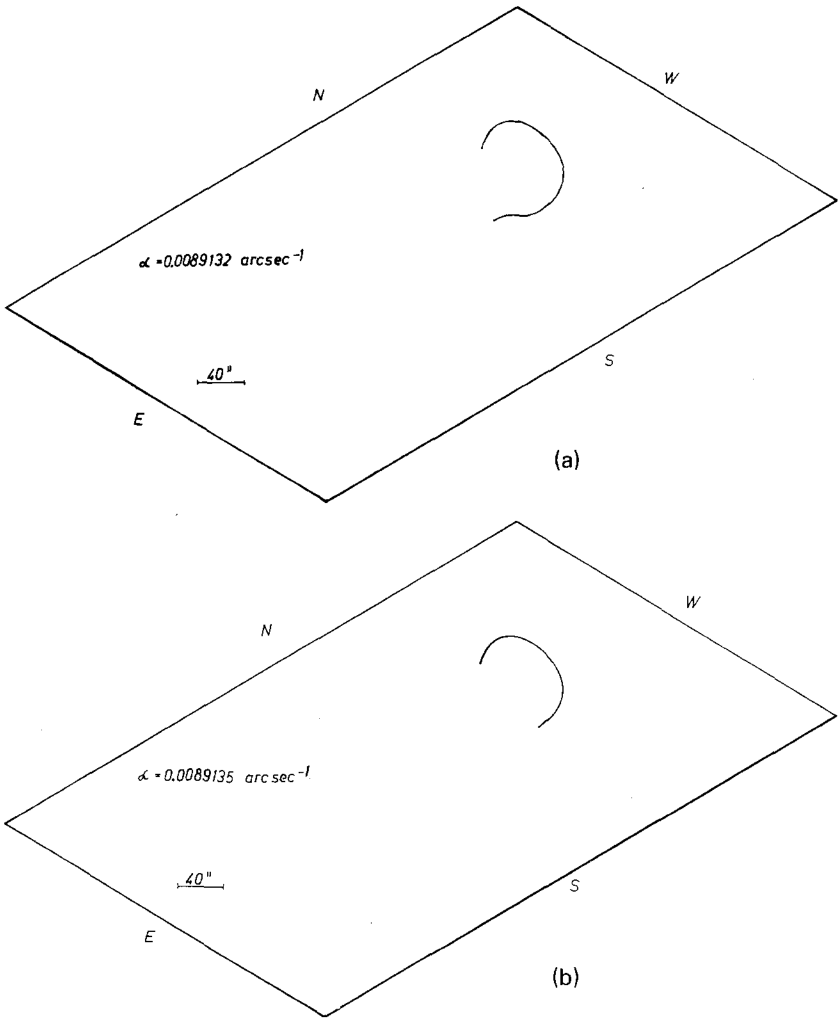
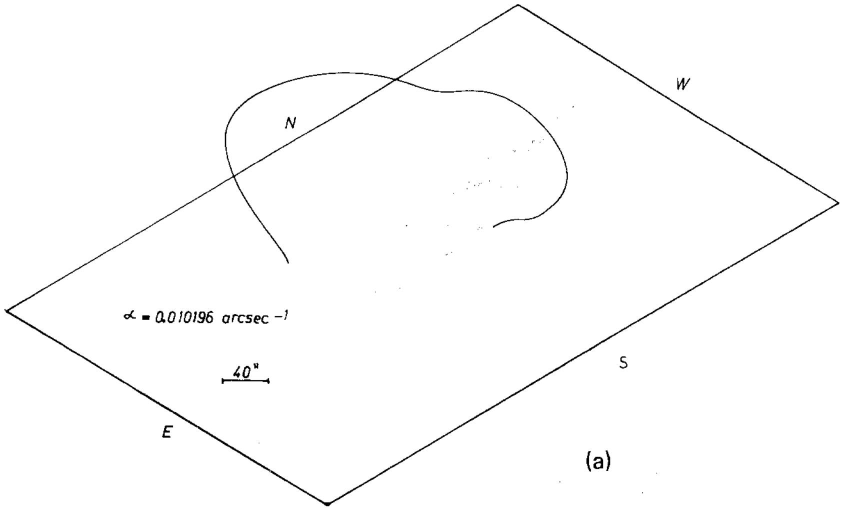


Fig. 2a–b. Field line traced from the point marked by a cross in Figure 1 for two values of the parameter α.

the field line starting point is kept fixed (and the photospheric normal field is kept fixed) the field line end point (on the photosphere) describes a curve which is discontinuous, i.e., has a gap, for a critical value $\alpha = \alpha_c$. The two parts of this curve are drawn in Figure 1. That part corresponding to α values less than α_c lies in the negative polarity of HR 16863 (with α increasing the end point moves from N to S on the curve), that for $\alpha > \alpha_c$ in the negative polarity of HR 16862. For $\alpha = \alpha_c$ the point jumps on the zero line of the normal field. How this takes place is illustrated in Figure 2(a–b), where the field line is drawn for two α values close to α_c.

The field line flow above the photosphere is partitioned into cells within which the field line mapping is continuous. The lines of intersection of the separatrix surfaces with the photospheric plane are, to some extent, visible in drawings of sets of randomly selected field lines, e.g., Figure 2(a) of Seehafer (1985a). They appear as strips not



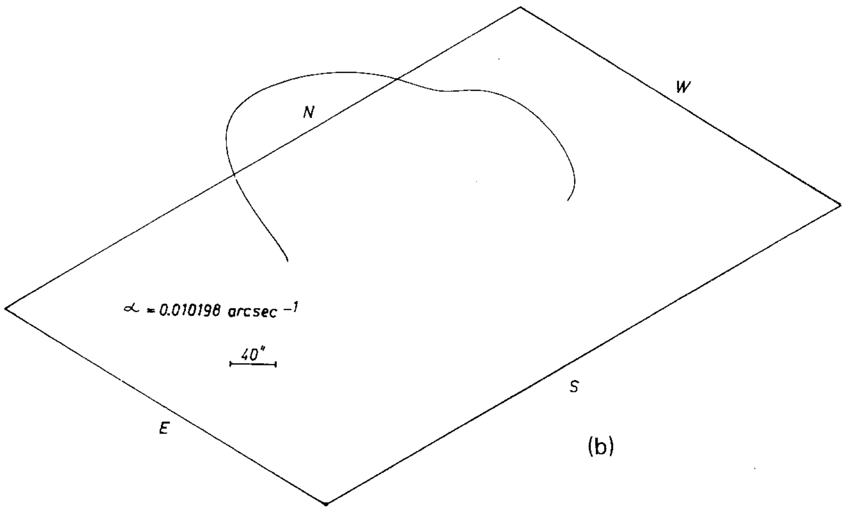


Fig. 3a–b. Field line traced from a photospheric starting point in the positive polarity of HR 16863 for two values of the parameter α.

covered by field lines. All major flux concentrations share their flux among different cells. The field structure is complicated by various small magnetic islands and the presence of lateral boundaries (cf. Seehafer, 1978).

The lines separating the photospheric flux have segments in common with the zero line of the photospheric longitudinal field. On such common line segments the transversal field is directed from negative to positive fields. The separatrix surfaces are made up of field lines which all touch the photosphere on one of these segments.

The global topology is rather involved. Figure 3(a–b) illustrates flux sharing, respectively, the corresponding transition from one cell to another with variation of α, in the positive polarity of HR 16863. The field lines on the separatrix surfaces are channeled on the line segments discussed.

5. Discussion

Syrovatskii (1981; see also Vasyliunas, 1975; Sonnerup, 1979) stresses the importance of field lines common to four different flux systems (X-lines, separators) for current sheet formation and magnetic field line reconnection. Such field lines are the lines of intersection (or self-intersection) of separatrix surfaces. They can also be expected for general field line configurations without singular points, as lines of intersection of separatrix surfaces made up of quasi-singular field lines. Then a separator does not connect between two singular points, as in the model of Sweet (1958), but is at one end or both ends tangential to the photosphere. For a simple four-cell topology due to two bipolar spot pairs the separator must touch the photosphere at both ends. If at one end point the photospheric normal field component would be different from zero, this point would lie in the interior of a photospheric flux concentration and would be common to all four flux cells. It cannot, however, be common to more than two flux cells because each photospheric flux concentration shares its flux among only two flux concentrations of opposite sign.

Note that flux sharing and separatrix surfaces are defined with respect to one definite surface, the photosphere. E.g., for the field generated by four point charges in a plane below the photosphere the separator with respect to the photosphere, which is (in general) the line of intersection of separatrix surfaces made up of quasi-singular field lines, is not identical with the field line connecting between the two saddle points in the subphotospheric plane (which cannot be tangential to the photosphere).

A change of the field line connectivity to the photosphere may be defined as magnetic reconnection. When the electrical conductivity of the plasma is infinitely high the field is frozen-in. Then, since the motion of the medium is a continuous deformation, the magnetic field evolves through topologically equivalent states. Two fields are topologically equivalent if there is a one-to-one continuous mapping with continuous inverse of the volume considered onto itself such that the field lines of the one field are transformed into those of the other.

A local change of the topology is possible only at singular points (cf. Seehafer, 1986). A special effect is introduced by the presence of boundaries which are not magnetic surfaces. Obviously transitions such as those displayed in Figures 2 and 3 are brought about by continuous deformations of the field line flow if the whole field above and below the photosphere is considered (and has a smooth continuation across the photosphere, which is the case for the force-free field model used). The transitions lead to non-equivalent topologies, however, if topological equivalence is defined with respect to the volume above the photosphere alone.

With such a separate consideration of the volume above the photosphere, qualitative topological changes require either magnetic field diffusion or exchange of material between the photosphere and the upper atmosphere. Both processes are slow with respect to the Alfvén time-scale of the upper atmosphere. For processes on the Alfvén time-scale, because of its high density, the photosphere can to some extent be considered as a rigid wall for the upper atmosphere. The anchoring of field lines in the photosphere (line-tying) imposes a topological constraint stabilizing the field against ideal mhd instabilities (Raadu, 1972; Hood and Priest, 1979; Van Hoven, 1981; Einaudi and Van Hoven, 1983), which are processes preserving the field topology (qualitatively) and proceeding on the fast Alfvén time-scale.

Mok and Van Hoven (1982) found that line-tying stabilizes also against resistive mhd instabilities. These – developing on a hybrid time-scale between the Alfvén and the diffusive time-scale – are still fast compared with pure magnetic field diffusion. Both ideal and, in particular, resistive mhd instabilities are used to explain the explosive release of magnetic energy in solar flares (Van Hoven, 1981).

With any evolution of the field for which diffusion is not negligible, there will be a continual passage of field lines through separatrix surfaces. As long as the large-scale field evolves through equilibrium states in response to photospheric motions on different scales, field line jumping occurs at a low rate. In order that the field can always relax to a constant-α force-free state, the whole evolution must be slower than this relaxation, which is assumed to involve small-scale magnetic reconnection (cf. Taylor, 1974; Heyvaerts and Priest, 1984).

Now consider a loop structure, or an arcade or any multi-loop structure, containing an amount of free magnetic energy sufficient to produce a flare and being stabilized against fast current-driven instabilities by line-tying. Let the loop be situated fully in one cell and approach a separatrix surface, or a separatrix surface approach the loop, respectively. Then the field lines of the loop jump successively and the stabilizing effect of line-tying is reduced.

Note in particular that when a loop has two (or more) parts situated in different cells, it can become subject to resistive instabilities, the essential feature of which is field line reconnection (White, 1983). As long as the loop remains situated fully in one single cell its topology cannot change and resistive instability is excluded. This is consistent with the result of Mok and Van Hoven (1982) that resistive instability of a finite-length cylindrical pinch with line-tying at the ends requires a reversal of the axial field component. The cylindrical surface on which the axial field vanishes is a degenerate separatrix surface.

Let, as in the example considered by Seehafer (1985a), the slow evolution be accompanied by an input of energy into the magnetic field. Then the following two developments are conceivable.

A fast instability can set in after only a small part of the loop field lines has passed through the cell boundary. Since the assumed fast energy dissipation cannot be compensated for by the slow energy input from the photosphere, the field lines which have jumped jump back. In such a situation the main part of the loop stays in the original cell, while continual energy input into the magnetic field leads to some heating.

Alternatively, unstable perturbations may exist only after a substantial part of the loop has passed into the new topology. Then the instability can lead to a flare.

The flares in the activity complex of May 1980 considered by Seehafer (1985a) are a possible example of flare initiation in this way. Another preferred flare site in the same complex was a small positive magnetic island north–west of the large leading (negative polarity) spot of HR 16864, indicated by an arrow in Figure 1, which formed a delta configuration with the main spot. The flare productivity of this configuration can be attributed to the western island boundary, part of which is a critical line segment of the kind discussed. (Seehafer, 1985b; the term 'neutral sheet' used in that paper should now be replaced by separatrix surface.) It is characteristic of delta configurations consisting of a parent spot and a satellite spot, compared with configurations with a parent spot and a satellite of the same polarity within a common penumbra, that the common penumbra is observable only between parent spot and satellite, but not at the outer satellite boundary, away from the main spot. This suggests that the magnetic flux from the satellite is (nearly) completely connected to the main spot and that a separatrix surface touches the photosphere at the penumbra-free part of the satellite boundary, a possible explanation for the flare productivity of these configurations (cf. Seehafer, 1985b). A similar example (no delta configuration), with all magnetic flux from a small flare productive magnetic feature connected to a central large spot of opposite polarity, was found by Seehafer and Staude (1980).

Concluding it should be noted that a definite decision whether magnetic neutral points exist or not will in general, if at all, only be possible after systematic numerical calculations, e.g., using index evaluation methods (Hsu and Guttalu, 1983). Thus it needs further studies to clarify whether perhaps structural bifurcations at neutral points (cf. Seehafer, 1986), one possibility for topological changes not due to the presence of boundaries, can play a role in solar active regions.

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