A COMPARISON OF DIFFERENT SOLAR MAGNETIC FIELD EXTRAPOLATION PROCEDURES

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Abstract. In order to study the influence of the different treatment of the boundary conditions in different extrapolation schemes, the procedures of Schmidt (1964), Nakagawa and Raadu (1972), and Seehafer (1978) have been applied to the same (line-of-sight) magnetogram. The main field structure is similar for all three procedures, whereas in details there are clear differences, for example in the direction of the field lines in the overview, the heights to which the field lines extend, the number of field lines that leave the region, the field strength decrease with height, and the calculated amounts of magnetic energy in the chromospheric and coronal parts of the considered active region.

1. Introduction

Presently reliable magnetic field measurements are restricted to the photospheric line-of-sight component. The magnetic field (vector) in the chromosphere and in the corona is generally calculated assuming the field to be current-free or force-free with constant α (of the equation $\nabla \times \mathbf{B} = \alpha\mathbf{B}$) and using the photospheric measurement (magnetogram) as boundary condition. If one determines current-free or constant-α force-free magnetic fields from magnetograms of limited photospheric regions, the data are not sufficient to define a unique boundary value problem (Seehafer, 1978). The possible influence of magnetic fields surrounding the magnetogram area must be referenced to in terms of additional assumptions or by specifying additional boundary conditions.

In order to study the influence of the different treatment of the boundary conditions in different extrapolation schemes that start from magnetograms of limited photospheric regions, the current-free procedure of Schmidt (1964, hereafter referred to as SCHP) and the constant-α force-free procedures of Nakagawa and Raadu (1972, referred to as NRP) and Seehafer (1978, referred to as SEEP) have been applied to the same magnetogram.

Differences between SCHP and NRP have already been discussed by Levine (1975), the relation between NRP and SEEP by Seehafer (1978).

2. Data and Calculations

The used magnetogram (Figure 1) was obtained at the Einstein tower telescope in Potsdam on 4 July 1973, with a resolution of $7\overset{''}{.}5 : 6\overset{''}{.}0$ and a scan area of $139'' : 143''$ in the EW : NW directions, respectively. The value of the longitudinal field averaged over the magnetogram area is -88 G. The region is characterized by a large sunspot of $p(-)$

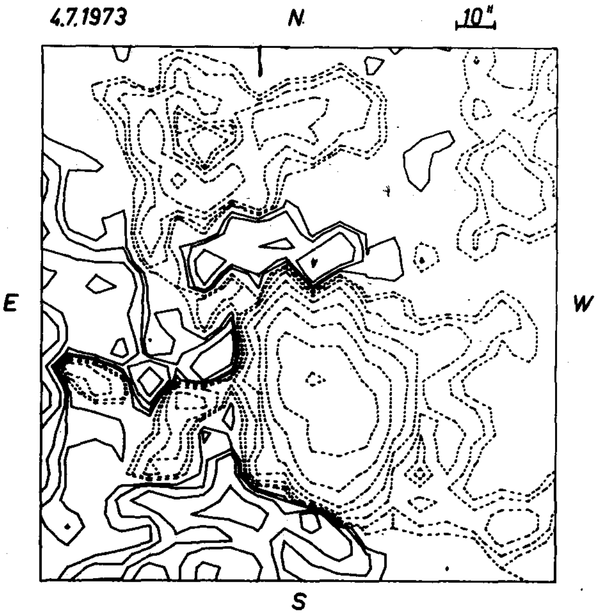

Fig. 1. Photoelectric magnetogram ($B_{\parallel}$) obtained at the Solar Observatory Einsteinturm in Potsdam on 4 July 1973, 08 : 30 UT. Contour levels are 20, 40, 80, 160, 320, 640, 1280, and 2560 G. Solid contours are (+) fields, dashed (−).

polarity (towards S from the centre in Figure 1). In connection with the elongated feature of $f(+)$ polarity in the north of the spot, results of extrapolation using SEEP have already been published (Seehafer and Staude, 1980): at the northern boundary of this feature an X-type neutral sheet is situated.

For all three procedures field lines have been calculated, starting from a mesh of photospheric foot points every $10'' : 10''$, in the case of the two force-free procedures for several values of the parameter α. Furthermore, the height dependence of the maximum field strength and the magnetic energy content of the (atmospheric part of the) region have been calculated.

The measured line-of-sight component of the magnetic field has been considered as the component normal to the (plane) photospheric boundary. In principle, the three procedures can be generalized such that they start from an oblique line-of-sight component (see Semel (1967) and Sakurai (1982) for SCHP, Wellck and Nakagawa (1973) for NRP, and Seehafer and Staude (1983) for SEEP). However, the considered active region (McMath 12417) is close to the center of the solar disk, and, furthermore, we are seeking for similarities and differences between the methods that do not depend on slight modifications of the input data.

3. Results

Figures 2, 3, and 4 show the lines of force of a potential field calculated according to the three procedures ($\alpha = 0$ for the two force-free procedures) in overview, perspective,

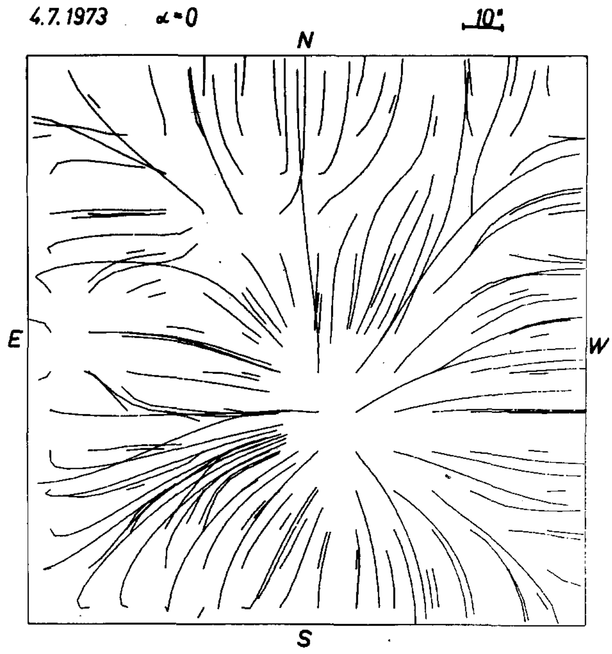

Fig. 2a.

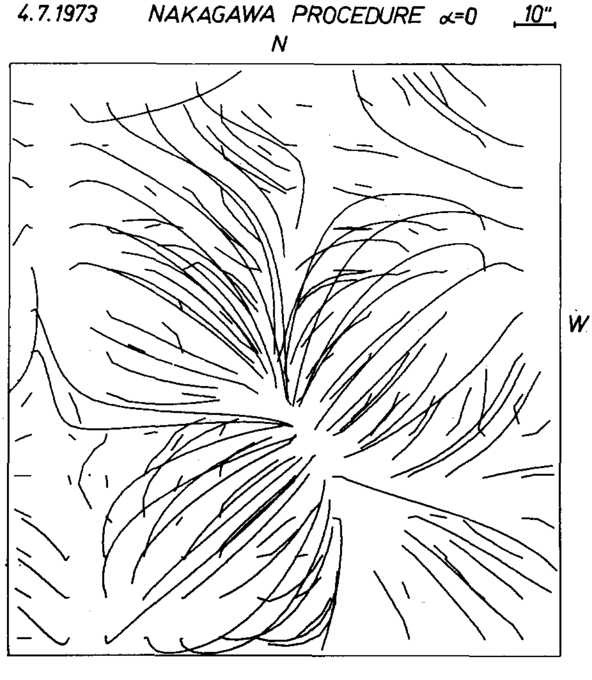

Fig. 2b.

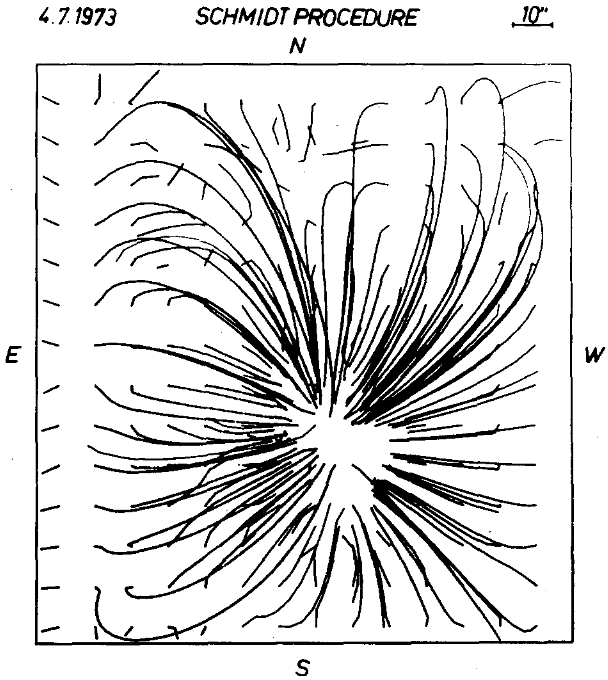

Fig. 2c.

Fig. 2a–c. Overview of the lines of force above the region of Figure 1 for a potential field calculated according to the Seehafer (a), Nakagawa-Raadu (b), and Schmidt (c) procedures.

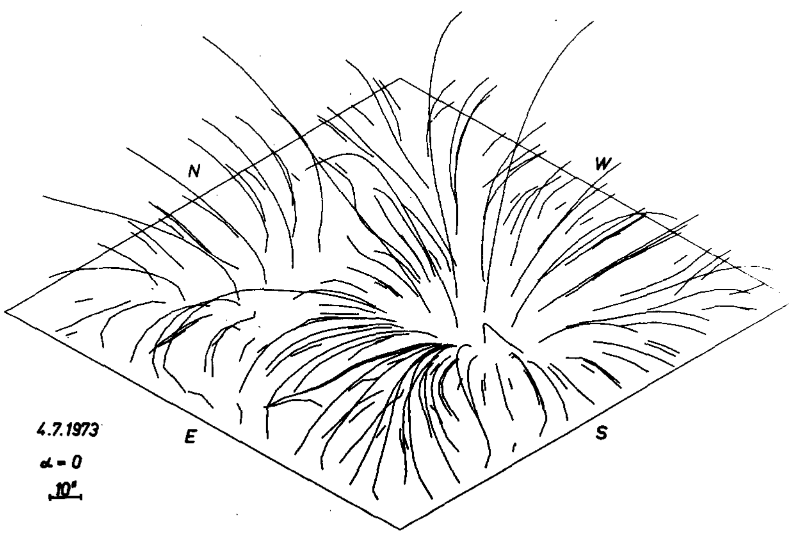

Fig. 3a.

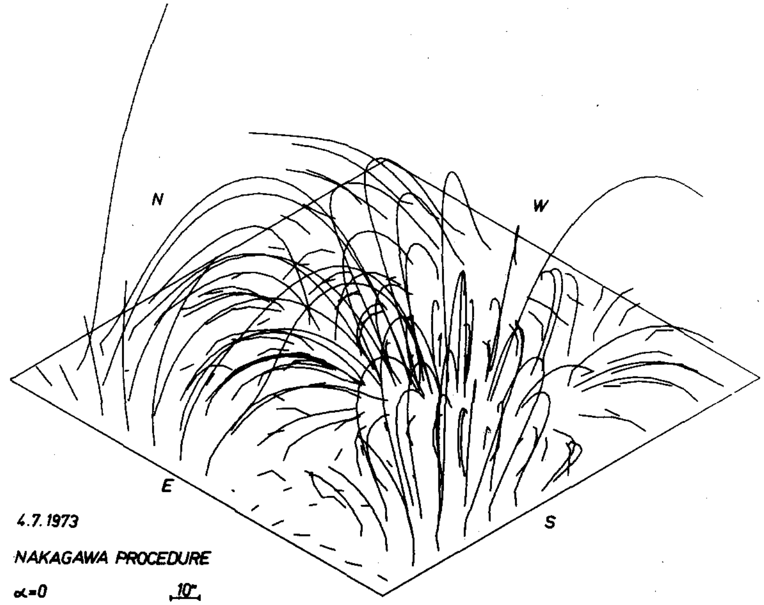

Fig. 3b.

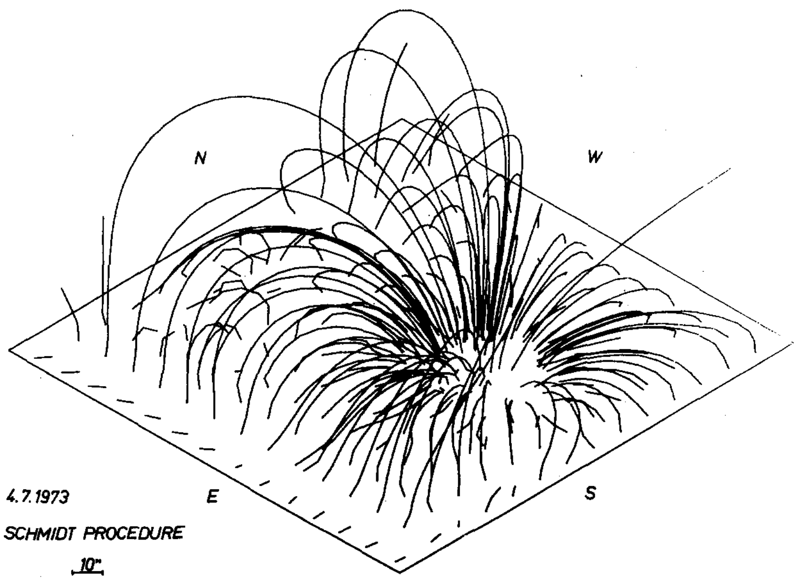

Fig. 3c.

Fig. 3a–c. Perspective view of the same calculated lines of force as in Figure 2.

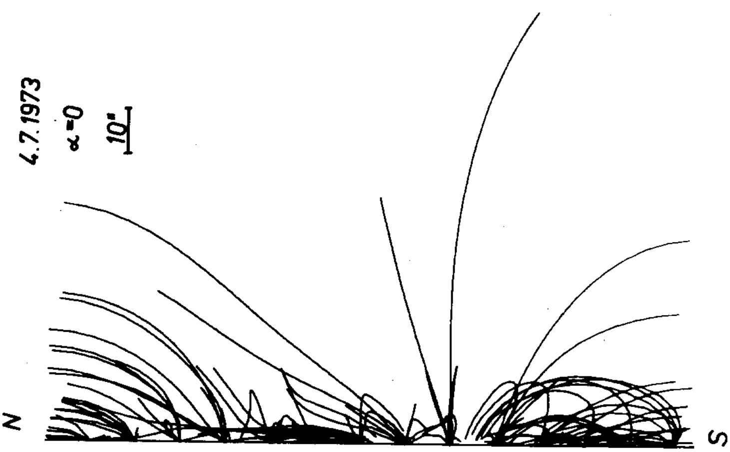

Fig. 4a.

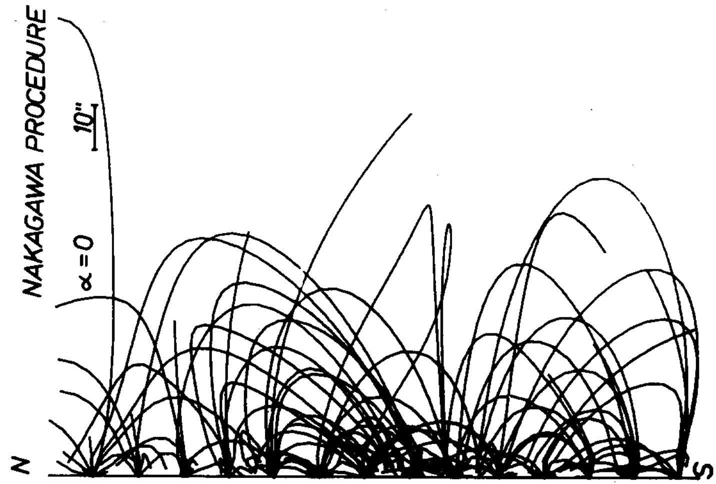

Fig. 4b.

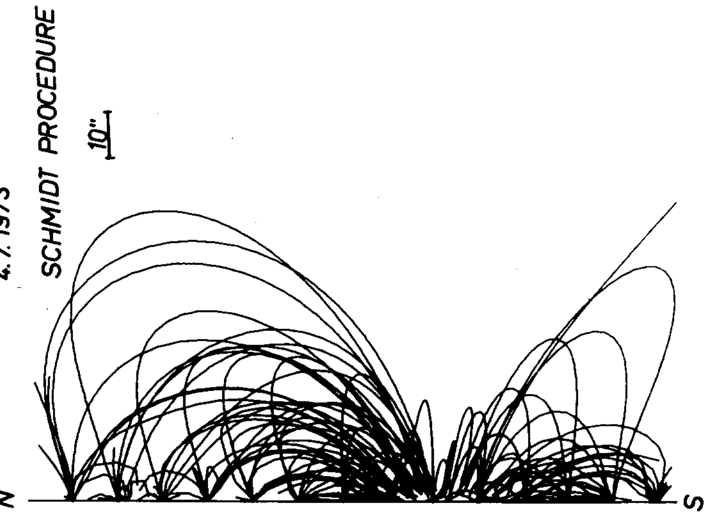

Fig. 4c.

Fig. 4a–c. Side view of the same calculated lines of force as in Figure 2.

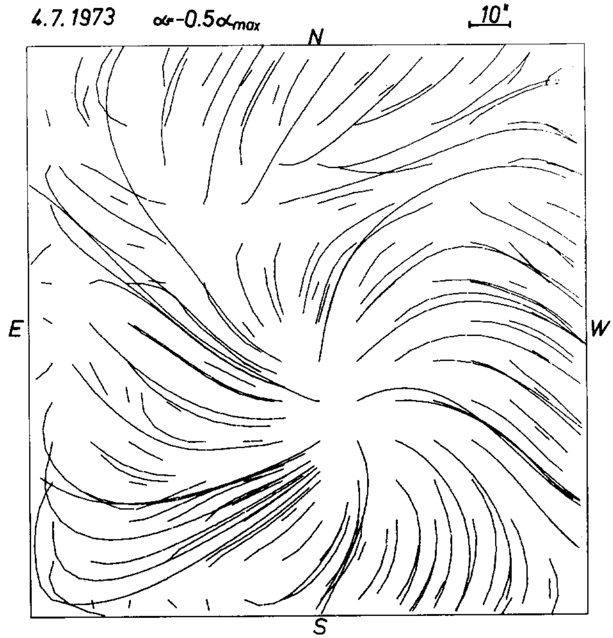

Fig. 5a.

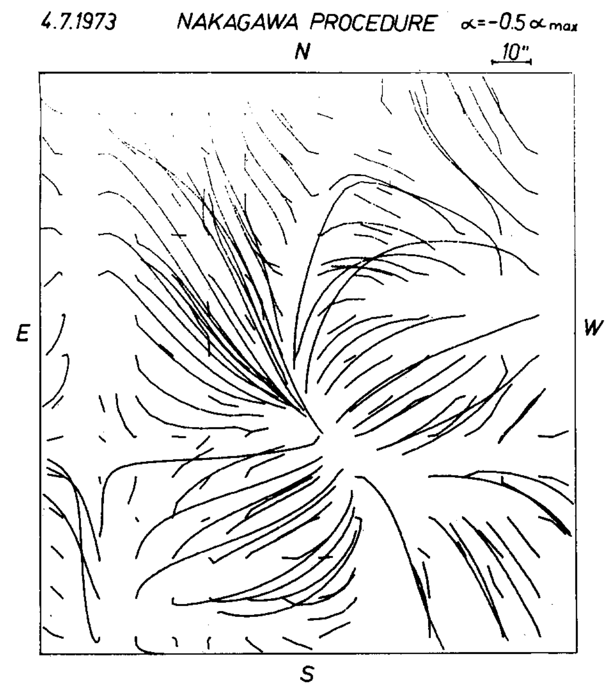

Fig. 5b.

Fig. 5a–b. Overview of the field lines above the region of Figure 1 for a (non-potential) force-free magnetic field calculated according to the Seehafer (a) and Nakagawa-Raadu (b) procedures.

and side view; Figure 5 shows field lines calculated according to the two force-free procedures for $\alpha = -0.5\alpha_{max}$ (the optimum value of α by comparison with Hα fibrils), where $\alpha_{max}^2 = \pi^2(L_x^{-2} + L_y^{-2})$, L_x and L_y being the extents of the magnetogram in both directions. The main field structure, which is dominated by the large spot, is similar for all three procedures. On the other hand, in details there are differences, for example in the direction of the field lines in the overview (which is generally compared with Hα, EUV, and X-ray structures).

The domain of analysis has been the rectangular straight cylinder above the magnetogram area. For SEEP significantly more field lines leave this volume through the sides than for NRP and SCHP (look at the northern boundary in Figure 3), for SCHP the number of such field lines being yet less than for NRP. Besides that, field lines calculated according to SCHP and NRP reach greater heights than those calculated using SEEP.

There is no simple correlation between the heights reached by the field lines and the field strength decrease with height, i.e., a higher reaching of the field lines is not necessarily connected with a slower decrease of the field strength. The height dependence of the maximum field strength calculated using the three procedures is displayed in Figure 6. For NRP the field strength is significantly less than for SEEP and SCHP, for SCHP somewhat less than for SEEP.

In Table I the magnetic energy contents of the region calculated according to the three procedures are compared. The differences are drastic, amounting to a factor of 10 between NRP and SCHP.

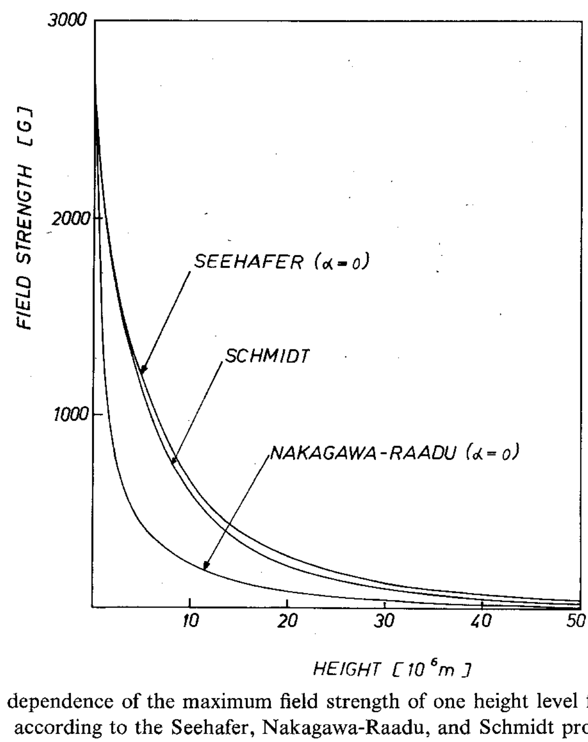

Fig. 6. Height dependence of the maximum field strength of one height level for a potential calculated according to the Seehafer, Nakagawa-Raadu, and Schmidt procedures.

TABLE I

Magnetic energy (E) in 10^{32} erg

$\frac{\alpha}{\alpha_{max}}$	0	0.5	0.9
E (SEEP)	6.1	6.5	9.3
E (NRP)	2.2	2.3	2.5
E (SCHP)	19.3	–	–

4. Discussion

From a comparison of SCHP and NRP Levine (1975) concluded that prohibiting field lines from leaving the volume through the sides causes them to become longer and higher. He found that field lines calculated using SCHP tend to extend higher and to start and to end on the magnetogram area. In fact, for SCHP field lines starting from the magnetogram must return to it. This is because for this procedure the actual volume for which the used solution holds is the half space above the plane of magnetograph observation, the field component normal to the boundary plane vanishing everywhere outside the magnetogram area itself. In the present analysis also for SCHP, field line tracing has been terminated at the sides of the rectangular column above the magnetogram area. For this reason in Figures 2c, 3c, and 4c some field lines seem to leave the region.

Also in the example presented here, for NRP, which allows for field lines starting from the magnetogram area without returning to it, some more field lines leave the volume than for SCHP, however yet much less than for SEEP. This is due to the fact that both SCHP and NRP assume the net magnetic flux through the magnetogram area, which must be connected with flux outside this area, to be zero, which SEEP does not. SCHP and NRP neglect the net flux through the magnetogram area, i.e., they do not start from the real magnetogram but from one which is got from the real one by subtracting the mean (vertical) field value, -88 G in the considered case. This value is relatively large and one may ask if SCHP and NRP are applicable, but for a methodical study such a magnetogram is well suited.

Contrary to NRP, for SCHP flux balance over the magnetogram area is not explicitly required. Indeed, SCHP is mathematically applicable also to cases of flux imbalance. Then the magnetic field vanishes at infinity only as r^{-2} (r denoting the distance from an origin), and the net flux through the magnetogram area is connected to infinity (the magnetic flux through an infinitely distant half sphere above the photospheric plane is not zero, but equal to the total flux through the magnetogram area). This, however, is a nonphysical feature. SCHP must be considered as an approximate (curvature-neglecting) calculation of a potential field exterior to a spherical photosphere. If such a field is represented by a spherical-harmonic expansion, for $\operatorname{div}\mathbf{B} = 0$ to be satisfied, the zero-order (monopole) term must be excluded. This means that the field vanishes at infinity at least as r^{-3}. In the present application of SCHP the net flux through the magnetogram area has been neglected. If this is not done, instead of directly applying the formulae of Schmidt (1964), the excess flux should be balanced by appropriately placed flux outside the magnetogram area in the photospheric plane and the necessary integration carried out over a correspondingly larger area. As to the comparison of SCHP with SEEP and NRP, it should be noted that for the two force-free procedures the situation is quite different insofar as in cases of $\alpha \neq 0$ for the boundary value problem to be unique and the magnetic energy to be finite (Chiu and Hilton, 1977; Seehafer, 1978; Alissandrakis, 1981) only solutions in the semi-infinite cylinder above the magnetogram area are considered.

Of course, field lines leaving the studied volume must be considered with caution. However, field lines staying inside the volume then also do not necessarily have a conclusive meaning. Whether or not a field line leaves the volume depends on the applied boundary conditions, and the staying inside of a field line may have been caused by boundary conditions which differ from those on the Sun. Thus prohibiting field lines from leaving the studied volume does not increase the reliability of an extrapolation procedure.

SEEP does not assume flux balance over the magnetogram area. The magnetic field is calculated from the magnetogram and the condition that the vertical field component vanishes on the sides of the considered rectangular cylinder, i.e., the condition that the field is horizontally directed there. By these boundary conditions the information contained in the magnetogram on those field lines leaving the volume becomes usable to some extent. It is not so that this is reached by replacing the flux balance condition of NRP by the condition on the vertical field component at the vertical boundary of SEEP. NRP, in addition to the flux balance condition, implies a condition at the vertical boundary as restrictive as that of SEEP (Seehafer, 1975, 1978). For both procedures these conditions at the vertical boundary may be expressed by requiring the Fourier expansions representing the two solutions to be twice differentiable term by term. The difference between SEEP and NRP results from the different choice of the set of eigenfunctions in which the solutions are expanded. A further eigenfunction-expansion solution, for the case of the potential field, was given by Teuber *et al.* (1977). They require the normal field component to vanish at the vertical boundary and, consequently, flux balance over the magnetogram area.

The eigenfunction-expansion solutions, which all imply conditions at the vertical boundary, imply also conditions at the boundary of the magnetogram. Before applying SEEP, for example, therefore a strip with vanishing vertical field component was added to the magnetogram. Without such a priori modifications there is (in general) a deviation of the calculated solution from the measured values at the boundary of the magnetogram.

The observed differences between the three procedures in the direction and location of field lines are in part due to the fact that SCHP and NRP start from a fictive magnetogram. Compared with the real magnetogram the zero lines are displaced and with them field lines bridging over them.

With increasing horizontal scale length of the magnetic field the field strength decrease with height will become slower. The horizontal scale length of the solution used in SEEP is larger than that of the solution used in NRP (Seehafer, 1978), that corresponding to SCHP being yet larger (infinite). Thus in Figure 6 the curve for SEEP should lie between those for NRP and SCHP. That this is not the case is due to the fact that NRP and SCHP start from a magnetogram in which the absolute value of the maximum vertical field strength is reduced by 88 G. This leads to reduced total vector field strengths also above the magnetogram plane. For magnetograms with small mean (vertical) field values the curve for SEEP should become located below that for SCHP.

As to the calculations of the magnetic energy content of the region (Table I) it must be mentioned that for SCHP the volume of calculation is not, as for NRP and SEEP, the rectangular cylinder above the magnetogram, but the whole half space in which the used solution is valid (Equation (4) of Schmidt (1964)). This is the only reason for the magnetic energy corresponding to SCHP being so large compared with the energies corresponding to SEEP and NRP, since from the field strength decrease above the magnetogram (Figure 6) for SCHP an energy less than that for SEEP would follow.

One may ask if also for SCHP only the magnetic energy in the column above the magnetogram area should be considered. However, for a separation of different parts of the magnetic energy to have a physical meaning, it must be based on information on the electric currents causing the magnetic field, which is not available.

In this context it should be noted that the physical meaning of the energy content of a potential field above the photosphere is not very clear, since the generating electric currents flow below the photospheric level and a separation of magnetic energies above and below this level is not justified.

The excess energy (compared with the potential field) of a force-free magnetic field above the photosphere, on the other hand, has a clear physical meaning, since it is due to atmospheric electric currents. For both SEEP and NRP the lateral boundary conditions are such that for the magnetic field to vanish at infinity the absolute value of the parameter α must be less than a maximum value, for SEEP this maximum value being somewhat smaller than for NRP. In Table I α_{max} denotes the maximum value for SEEP. As $|\alpha|$ approaches the maximum value the magnetic energy becomes infinite. This explains why with $|\alpha|$ increasing the ratio of the magnetic energies corresponding to SEEP and NRP increases. On the other hand it shows that the boundary conditions may have a greater influence on the excess energy stored than the value of α.

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