FORCE-FREE EQUILIBRIUM AT MAGNETIC NEUTRAL POINTS

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Abstract. It is shown that, at neutral points of force-free magnetic fields, the electric current density must vanish. This property is independent of whether the neutral points are isolated or (e.g.) fill lines or surfaces. One implication is the fact that in a cold pressureless plasma the formation of neutral current sheets cannot be adiabatically slow. The field-line topology in the neighbourhood of neutral points is discussed. At neutral points of force-free magnetic fields in general three constant-α surfaces, α defined by the equation $\nabla \times \mathbf{B} = \alpha \mathbf{B}$, with the same value of α intersect orthogonally. If, during a time-development, the magnetic field gradient matrix $\partial B_i/\partial x_j$ becomes singular at a neutral point, the field topology can change qualitatively – in general connected with the merger of two or more neutral points into one and/or the splitting up of one neutral point into several others. This can be interpreted as implying the transition from a quasi-static evolution to a dynamical state in which magnetic energy is released.

1. Introduction

Neutral (singular, null) points of magnetic fields, where the field vanishes, have received much astrophysical interest since they are believed to be the preferential sites of the explosive conversion of magnetic energy into particle energies. It is widely accepted that solar flares and magnetospheric substorms are manifestations of such an energy conversion. Probably the mechanisms studied there are relevant for flare stars, maybe also for other astronomical phenomena such as radio galaxies and quasars (Sturrock and Knight, 1976).

In most models of the energy conversion process a cooperation between magnetic field convection and diffusion, accompanied by field line reconnection, is essential (see recent reviews by Vasyliunas, 1975; Sonnerup, 1979; Parker, 1979, Chapter 15; Baum and Bratenahl, 1980; Syrovatskii, 1981; White, 1983; Galeev, 1984). To get a sufficient amount of magnetic field diffusion for the whole process, the reconnection process, to operate fast enough, because of the high electrical conductivity of the plasmas encountered very steep current gradients are needed. Therefore the existence of current sheets (Syrovatskii, 1981; Priest, 1981) is assumed; an alternative model with tearing-mode reconnection in sheared loop structures has been proposed by Spicer (1977; see also review by Van Hoven, 1981).

One way in which a current sheet may form is the magnetic collapse of the region near a magnetic neutral point. This process, suggested by Dungey (1953), has been studied particularly by Syrovatskii and his co-workers (cf. review by Syrovatskii, 1981), most recently by Bulanov and Olshanetsky (1984) and by Bulanov *et al.* (1984). The time-developments considered start from an equilibrium state. Whether this is unstable and whether the nonlinear development of disturbances leads to the formation of a current

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sheet depends on the magnetic field configuration near the neutral point in the initial state, the kind of the disturbances, and the boundary conditions imposed. Hitherto in all explicit models the initial magnetic field was assumed to be current-free. However, a generalization to models with force-free magnetic fields – i.e., magnetic field aligned electric currents – would be useful, since the currents represent magnetic energy free to be released; whereas in potential field models the energy must be transported, by waves, from distant sources to the current sheet.

Force-free magnetic fields are common in (hot) tenuous cosmic plasmas such as stellar envelopes, in particular the solar atmosphere. There a low mass-density is connected with a correspondingly low energy-density of the plasma, the particle number density being on the other hand still high enough for a high electrical conductivity, so that Lorentz forces large compared with the non-magnetic forces are possible and in equilibrium significant electric currents must be aligned with the magnetic field.

In Section 2 of the present paper it is proven that in neutral points of force-free magnetic fields the electric current density must vanish. The implications of this result for the field structure in the neighbourhood of neutral points may help to clarify whether the presently discussed current sheet formation processes can work in or can be generalized to force-free fields. Moreover, it is important to know the possible types of the neutral points of a magnetic field (cf. Section 3) since positions and types of the neutral points determine the qualitative structure of the field.It has been suggested that changes of the field topology during a slow evolution through a sequence of equilibrium states cause the transition from the quasi-static evolution to a dynamical state with an explosive release of magnetic energy (Seehafer, 1985; reviews by Birn and Schindler, 1981; Low, 1982). Local changes of the field topology can be understood as type changes of a neutral point.

2. Current-Free Character of Neutral Points of Force-Free Magnetic Fields

In the low-frequency (mhd) approximation,

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B}\,, \tag{1}$$

a force-free magnetic field is defined by

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0\,, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0\,. \tag{3}$$

For any non-neutral point and its neighbourhood we can write

$$\nabla \times \mathbf{B} = \alpha(\mathbf{x}) \cdot \mathbf{B}\,, \tag{4}$$

with α denoting a (pseudo-) scalar function of position. $\alpha(\mathbf{x})$ can be assumed to be continuous and differentiable to any order if $\mathbf{B}(\mathbf{x})$ is assumed to be sufficiently smooth, since in the neighbourhood of the non-neutral point there is a non-vanishing field component B_i, and

$$\alpha = \frac{(\nabla \times \mathbf{B})_i}{B_i} . \tag{5}$$

From Equation (4) it follows that

$$\alpha = (\nabla \times \mathbf{t}) \cdot \mathbf{t} , \tag{6}$$

where $\mathbf{t}$ denotes the unit vector in the direction of $\mathbf{B}$, $\mathbf{B} = |\mathbf{B}| \cdot \mathbf{t}$. Thus $\alpha(\mathbf{x})$ is a simple function of the field line geometry, independent of the specification of $|\mathbf{B}|$ (cf. Boström, 1973). In neutral points $\mathbf{t}$ and accordingly α are not defined.

For fields with spatially constant α, the so-called constant-α or linear (because with α prescribed the field is determined from a linear equation) force-free fields (cf., e.g., Seehafer, 1978), a neutral point of the $\mathbf{B}$-field is obviously also a neutral point of the $\mathbf{j}$-field.

Now assume that, for a $\mathbf{B}$-field with non-constant α, $\mathbf{x}_0$ is a neutral point of the $\mathbf{B}$-field and a non-neutral point of the $\mathbf{j}$-field: $\mathbf{B}(\mathbf{x}_0) = 0, \mathbf{j}(\mathbf{x}_0) \neq 0$. Then in any neighbourhood of $\mathbf{x}_0$ there are non-neutral points of the $\mathbf{B}$-field since otherwise $\mathbf{j}(\mathbf{x}_0) = 0$.

Let $\hat{\mathbf{t}}$ denote the unit vector in the direction of $\mathbf{j}$, $\mathbf{j} = |\mathbf{j}| \cdot \hat{\mathbf{t}}$, and $\hat{\alpha}$ be defined by

$$\hat{\alpha} = (\nabla \times \hat{\mathbf{t}}) \cdot \hat{\mathbf{t}} . \tag{7}$$

It is given by

$$\hat{\alpha} = \frac{(\nabla \times \mathbf{j}) \cdot \mathbf{t}}{|\mathbf{j}|} . \tag{8}$$

Assume that $\mathbf{j}$ is differentiable with bounded derivatives (respectively, that $\mathbf{B}$ is twice differentiable with bounded second-order derivatives). Then in $\mathbf{x}_0$ and a (closed) neighbourhood of $\mathbf{x}_0$ $\hat{\alpha}$ is well-defined and bounded. Consider a sequence of non-singular points of the $\mathbf{B}$-field which tends to $\mathbf{x}_0$. Then α tends to infinity, as can be seen from Equation (4), whereas $\hat{\alpha}$ is bounded (α is defined in the points of the sequence since because of the assumed continuity of $\mathbf{B}$ each of these points has a neighbourhood in which $\mathbf{B} \neq 0$). This, however, is a contradiction, since for $\mathbf{j} \neq 0$ and $\mathbf{B} \neq 0$, that is for all points of the sequence, $\mathbf{t} = \hat{\mathbf{t}}$ and $\alpha = \hat{\alpha}$.

Thus we have established that, at a neutral point of a force-free magnetic field, the electric current density must vanish.

3. Field Line Topology Near Neutral Points

Parametric representations $\mathbf{x}(t)$ of individual magnetic field lines are obtained as solutions of the differential equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{B} . \tag{9}$$

The entity of the field lines is called field line flow, analogously to the phase flow of a dynamical system, which denotes the entity of the phase space trajectories. The singular points of $\mathbf{B}$ are the fixed points of the field line flow and vice versa (Arnold, 1973). Field lines can only approach singular points as $t \to \infty$ or $t \to -\infty$. The classification of singular points of (two-dimensional) vector fields can be found in books on nonlinear oscillations (e.g., Minorsky, 1962; Bogoliubov and Mitropolsky, 1961). Neutral points of magnetic fields in particular have been studied by Dungey (1953), Stern (1966), Bulanov *et al.* (1984), and most carefully by Fukao *et al.* (1975). All these studies are based on the first-order representation of $\mathbf{B}$ in the neighbourhood of the neutral point $\mathbf{x}_0$,

$$\mathbf{B} = A(\mathbf{x} - \mathbf{x}_0)\,, \tag{10}$$

where A denotes the gradient matrix of $\mathbf{B}$,

$$A_{ij} = \frac{\partial B_i}{\partial x_j}\,, \qquad \mathbf{x} = \mathbf{x}_0\,. \tag{11}$$

Because of $\operatorname{div}\mathbf{B} = 0$ the trace of A is zero,

$$\operatorname{tr}(A) = 0\,. \tag{12}$$

Consider two vector fields $\mathbf{v}(\mathbf{x})$ and $\mathbf{w}(\mathbf{x})$ defined on a spatial region M, which is assumed to be a differentiable manifold, e.g., all space or an open subset of it. The field line flows of $\mathbf{v}$ and $\mathbf{w}$ are called topologically equivalent (and $\mathbf{v}$ and $\mathbf{w}$ topologically orbitally equivalent) if there is a homeorphism (one-to-one continuous mapping with continuous inverse) of M on itself such that the field lines of $\mathbf{v}$ are transformed into the field lines of $\mathbf{w}$ (while the field direction along the field lines is preserved).

Whith $\mathbf{B}$ given by Equation (10) and $\mathbf{x}_0 = 0$, Equation (9) takes the form

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = A\mathbf{x}\,. \tag{13}$$

Two such linear systems (respectively, the field line flows which they define) are topologically equivalent if their coefficient matrices have only eigenvalues with non-vanishing real parts and if the number of eigenvalues with positive (negative) real parts is the same for both systems (Arnold, 1973). In the neighbourhood of a fixed point a nonlinear system is topologically equivalent to its linear part if the linear part has only eigenvalues with real parts different from zero.

If, on the other hand, the matrix A given by Equation (11) has an eigenvalue with vanishing real part the use of the first-order representation of $\mathbf{B}$ is not sufficient for studying the qualitative field-line behaviour near the neutral point.

Because the current density vanishes at the magnetic neutral point, for a force-free field A is symmetric, that is its eigenvalues are real and the eigenvectors orthogonal.

If we exclude the case when one of the eigenvalues vanishes – i.e., assume $\det(A) \neq 0$, because of $\operatorname{tr}(A) = 0$ two eigenvalues are positive and one negative or two negative and one positive. That is near the neutral point $\mathbf{B}$ is topologically equivalent to a three-dimensional saddle: With the axes of an orthogonal Cartesian coordinate system given by the principal axes of A, the general solution of Equation (13) is

$$\mathbf{x} = \sum_{i=1}^{3} c_i \mathbf{x}_i e^{\lambda_i t}, \tag{14}$$

where the λ_i and $\mathbf{x}_i$ denote the eigenvalues and corresponding unit eigenvectors (unit vectors along the coordinate axes), respectively, and the c_i are arbitrary constants. If a field line is on a coordinate axis or on a coordinate plane, it does not leave this axis or this plane. Let λ_1 and λ_2 be positive and λ_3 negative. In the $x_1 - x_2$ plane all field lines tend to the origin as $t \to -\infty$, i.e., the origin is a node. The positive and the negative x_3 half-axes are field lines tending to the origin as $t \to +\infty$. In the $x_1 - x_3$ and in the $x_2 - x_3$ plane the origin is a saddle (X) point.

It should be noted that, in the general case when no real part of an eigenvalue of A vanishes, also non-force-free fields are topologically equivalent to a saddle in the neighbourhood of the neutral point, since this is a consequence of $\mathrm{tr}(A) = 0$. In particular in two dimensions a focus, where the field lines tend to the singular point while infinitely often spiraling around it (and which corresponds to a pair of complex conjugate eigenvalues), is topologically equivalent to a node.

From Equations (3) and (4) it follows that

$$(\nabla\alpha) \cdot \mathbf{B} = 0, \tag{15}$$

i.e., α is constant along a field line and the field lines lie on surfaces α = constant. For the three-dimensional linear saddle the coordinate planes are magnetic surfaces (the normal field component vanishes on them) and, therefore, surfaces α = constant. Since these planes have common field lines, namely the coordinate axes, the α value must be the same for all three planes.

For nonlinear **B** the planes are deformed to smooth surfaces composed of field lines (invariant manifolds). A neutral point of a force-free field with $\det(A) \neq 0$ is a point of intersection of three constant-α surfaces with the same value of α.

Much less can be said if $\det(A) = 0$. Because of $\mathrm{tr}(A) = 0$ then either one eigenvalue vanishes and two are different from zero or all three eigenvalues vanish.

Let one eigenvalue be zero and two different from zero. Then one can use the theorem (cf. Bogoyavlensky, 1980) that if k eigenvalues have negative real parts there is locally a k-dimensional invariant manifold on which all field lines tend to the singular point as $t \to \infty$ (for k eigenvalues with positive real parts correspondingly as $t \to -\infty$). In the case considered, where one eigenvalue is positive and one negative, this means that there are two smooth curves through the singular point such that one of the curves is a pair of field lines tending to the singular point and the other a pair field lines diverging from the singular point. α being not defined at the singular point, the values of α on the four field lines may be different from each other.

The two curves through the singular point intersect orthogonally, since when field lines have a definite direction at the singular point this must be the direction of an eigenvector of the linearized system (cf. Bogoyavlensky, 1980). Correspondingly, in the case of three nonvanishing eigenvalues the three constant-α surfaces intersect orthogonally, though if the two eigenvalues with equal sign have also equal absolute values the node in the $x_1 - x_2$ plane may be deformed to a focus.

4. Discussion

It should be noted that in deriving the result of Section 2, the current-free character of neutral points of force-free magnetic fields, no use has been made of $\operatorname{div} \mathbf{B} = 0$. Thus this result can be applied to hydrodynamic flows with parallel vortex lines and streamlines (Beltrami flows), no matter whether these are incompressible or compressible – i.e., whether or not the flow field is solenoidal.

Neutral points of non-force-free magnetohydrostatic equilibria, which are characterized by

$$\nabla \times ((\nabla \times \mathbf{B}) \times \mathbf{B}) = 0\,, \tag{16}$$

are not necessarily current-free. A simple counter-example is the cylindrically symmetric linear pinch with uniform axial current density, where $\mathbf{B}$ vanishes along the symmetry axis (cf. Shercliff, 1965; p. 71).

Using $\mathbf{B}(\mathbf{x}_0) = 0$ and Equations (1) and (3), at a magnetic neutral point $\mathbf{x}_0$, Equation (16) takes the form

$$(\mathbf{j} \cdot \nabla)\mathbf{B} = 0\,, \qquad \mathbf{x} = \mathbf{x}_0\,. \tag{17}$$

If $\mathbf{j}(\mathbf{x}_0) \neq 0$, $\mathbf{j}$ defines a direction at $\mathbf{x}_0$ and the derivatives of $\mathbf{B}$ in this direction vanish there. Sweet (1958) studied the field configuration near neutral points of magnetohydrostatic fields representing $\mathbf{B}$ by its linear terms and concluded that the neutral points must fill lines (or surfaces). In fact, in the linear approximation Equation (17) implies that $\mathbf{B}$ is invariant along the direction of $\mathbf{j}$, but nothing is said about the influence of the higher-order terms.

If we choose a coordinate system with one axis along the direction of $\mathbf{j}$, we see from Equation (17) that the gradient matrix of $\mathbf{B}$, defined by Equation (11), is non-regular; i.e., admits of the eigenvalue zero. Then the other two eigenvalues are both zero, both purely imaginary and complex conjugate, or both real with equal absolute values and opposite signs; the latter case is briefly discussed in Section 3.

An evident consequence of the current-free character of the neutral points is that in force-free magnetic fields neutral current sheets, where the magnetic field vanishes in the centre of the sheet, cannot exist and that in a cold pressureless plasma the formation of a neutral current sheet cannot be adiabatically slow.

Let the magnetic field evolve through a sequence of force-free equilibria. As long as no eigenvalue of the gradient matrix A changes sign these equilibria are topologically equivalent in the neighbourhood of the neutral point. If, however, one eigenvalue becomes zero, the transition to a non-equivalent topology, a structural bifurcation (Kubíček and Marek, 1983, Appendix C; Haken, 1983) can occur.

Bifurcations connected with the passage of real eigenvalues through zero are called real bifurcations. A complex bifurcation (Hopf bifurcation) occurs if a pair of complex conjugate eigenvalues crosses the imaginary axis. In a magnetic field this latter effect cannot be isolated, since because of $\mathrm{tr}(A) = 0$ the third eigenvalue, which is real, must vanish then.

Local qualitative changes of the field-line topology are possible only at neutral points of the field. In the neighbourhood of a given point $\mathbf{x}_0$ all vector fields for which $\mathbf{x}_0$ is a non-neutral point are topologically equivalent to each other: given a continuously differentiable vector field $\mathbf{v}(\mathbf{x})$ with $\mathbf{v}(\mathbf{x}_0) \neq 0$, for a sufficiently small neighbourhood M of $\mathbf{x}_0$ there is a diffeomorphism (one-to-one differentiable mapping with differentiable inverse) of M onto itself such that the field lines of $\mathbf{v}$ are transformed into straight lines (Arnold, 1973).

A local change of the field topology at a neutral point is obviously accompanied by (respectively, represents also) a global topological change of the field in which the neutral point is embedded. On the other hand neutral points are not necessary for global topological changes, in particular if infinitely long (including closed) field lines are present (cf. Haken, 1983).

So far the existence of magnetic neutral points has been presupposed. In general the state of the system considered will depend on external parameters (control parameters). Changes of these parameters induce (or describe, respectively) an evolution of the system. The loci of the neutral points are obtained as solutions $\mathbf{x}(\mathbf{a})$ of the equation

$$\mathbf{B}(\mathbf{x}, \mathbf{a}) = 0 , \tag{18}$$

where $\mathbf{a}$ denotes a vector of parameters. Given a pair $(\mathbf{x}_0, \mathbf{a}_0)$ such that Equation (18) is satisfied, according to the implicit function theorem, in a neighbourhood of $\mathbf{a}_0$ there is a unique solution $\mathbf{x}(\mathbf{a})$ if the Jacobian determinant of $\mathbf{B}$ with respect to $\mathbf{x}$, that is the determinant of the magnetic field gradient matrix A is different from zero at $\mathbf{x} = \mathbf{x}_0$, $\mathbf{a} = \mathbf{a}_0$.

Thus, since a purely complex bifurcation is not possible, for magnetic fields the condition for structural instability, $\det(A) = 0$, coincides with the condition for branch points in $\mathbf{x} - \mathbf{a}$ space (cf. Kubíček and Marek, 1983). At such branch points different solution curves $\mathbf{x}(\mathbf{a})$ (branches) meet or cross each other. Therefore, a structural bifurcation of $\mathbf{B}$ at a neutral point will in general be connected with the merging of two or more neutral points into one and/or the splitting up of one neutral point into several others. The origination of a closed field line (limit cycle) from a neutral point (which represents itself a field line), typical of a Hopf bifurcation, is not to be expected during the evolution of force-free fields, at least if not all three eigenvalues vanish.

In the general, structurally stable case, when all eigenvalues are different from zero, at a neutral point of a force-free magnetic field three magnetic surfaces intersect. Under a structural bifurcation these surfaces are destroyed, at least in part. In fusion research the existence and preservation of magnetic surfaces is considered as an essential requirement for plasma confinement by magnetic fields (Morozov and Solov'ev, 1966; White, 1983). Also in cosmic situations the destruction of magnetic surfaces may cause the transition of a quiescent plasma to a dynamical state.

Magnetic surfaces separate spatial regions with different plasma properties from each other. In solar active regions, for example, density and temperature are significantly increased compared with the rest of the solar atmosphere; besides that an active region is partitioned in subregions with different plasma regimes. Sweet (1958; see also Syrovatskii, 1981) has modelled the magnetic field of a region with four sunspots (two bipolar spot pairs) by a four-cell structure with two neutral points. The four cells are separated by magnetic surfaces and any field line on these surfaces ends or starts in at least one of the neutral points. Evidently a topological change at one of the neutral points destroys the whole equilibrium structure.

In a plasma of infinitely high electrical conductivity, due to the frozen-in-field condition structural bifurcations of the magnetic field are not possible. With a small amount of resistivity a quasi-static evolution including topological changes seems possible if the evolution is sufficiently slow; if fluctuations are present these can provide for the necessary decoupling of the mean magnetic field from the plasma.

On the other hand, a nearby but topologically non-equivalent state with lower magnetic energy can exist. If the system approaches a structurally unstable state a small disturbance connected with a change of the magnetic field topology may lead to the onset of a dynamical process during which free magnetic energy is released.

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