Exercise 14
Linear Algebra
Bonus tasks

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1. Task *Solving systems of linear equations* (10 extra P)
Let

$$\boldsymbol{A} = \begin{pmatrix} \frac{\pi}{3} & \frac{\pi}{3} & \dots & \dots & \dots & \frac{\pi}{3} \\ 0 & \frac{\pi}{3} & \dots & \dots & \dots & \frac{\pi}{3} \\ 0 & 0 & \frac{\pi}{3} & \dots & \dots & \frac{\pi}{3} \\ 0 & 0 & 0 & \ddots & \dots & \frac{\pi}{3} \\ 0 & 0 & 0 & \dots & \frac{\pi}{3} & \frac{\pi}{3} \\ 0 & 0 & 0 & & \dots & \frac{\pi}{3} \end{pmatrix} \in \mathbb{R}^{n\times n} \quad \text{and} \quad \boldsymbol{b} = \begin{pmatrix} n \\ n \\ \vdots \\ \vdots \\ n \\ n \end{pmatrix} \in \mathbb{R}^n \tag{1}$$

for different values of $n \geq 1000$.
Write a program (C/C++)that solves $\boldsymbol{Ax} = \boldsymbol{b}$ for $\boldsymbol{x}$ numercially

a) column-wise

b) row-wise

and measure the runtime of both versions with help of, e.g., `omp_get_wtime()`. Make n sufficiently large to get significant different runtimes. Explain the difference.
Hint: While the row-wise version might be straightforward to program (outer loop over first index i from $n-1$ to 1), the column-wise implementation (outer loop over second index j from $n-1$ to 1) looks in pseudo code like that:

```
for j = n-1 ... 1
  for i = 1 ... j
      b[i] = b[i] - a[i][j+1] * x[j]
  x[j] = b[j] / a[j][j]
```

2. Task *Nonlinear fit* (10 extra P)
Fit the Breit-Wigner formula

$$f(E) = \frac{f_\mathrm{r}}{(E - E_\mathrm{r})^2 + \Gamma^2/4} \tag{2}$$

via the parameters $f_\mathrm{r}, E_\mathrm{r}, \Gamma$ to the cross section data from the lecture. For this purpose write a program that fits the parameters with help of the Newton-Raphson method, i.e., by iteration and computing the Jacobian matrix. How sensitive the solution is to the initial guesses for a_1, a_2, a_3?
Plot the resulting function together with the data. Determine with the mentioned method E_r and Γ (width of the resonance).