Exercise 12

C/C++ Monte Carlo simulations, MC-Integration

(02.07.2025, hand in 09.07.2025)

1. Task *Probability distributions* (5 P)

a) Find the inverse transformation $x = P^{-1}(r)$ to get *uniformly distributed* random numbers x in the interval $[a, b]$ $(0 \leq r \leq 1)$. First, find $p(x)$. (1 P)

b) How can one get *normally distributed* random numbers z_i for $\sigma \neq 1$ and $\mu \neq 0$ from *standard normally distributed* x_i with $\sigma = 1$ & $\mu = 0$ as, e.g., obtained from the Box-Muller method? (1 P)

c) Write a program that uses the Box-Muller method to generate *normally distributed* random numbers. Show, i.e. plot a *histogram* (e.g., with `gnuplot`) together with a Gaussian bell curve, that these numbers are indeed normally distributed. (3 P)

2. Task *Inelastic neutron scattering* (4 P)
Now, let us consider the case of inelastic scattering, i.e., each time a neutron is scattered, it looses a fraction f of its energy E. Furthermore, the mean free path is $\lambda_{\text{mfp}} = \sqrt{E}$. Repeat the task 2.b) of exercise 10 with these modifications and compare the results for $f = 0.05,\ 0.1$ and 0.5.
Plot a histogram for the distributions of path lengths between two scattering events, each for $f = 0.1, 0.5$, and 0 (elastic scattering).

3. Task *Monte-Carlo integration* (4 P)
Determine the estimated value of the integral F_n for the

$$f(x) = 4\sqrt{1 - x^2} \tag{1}$$

in the interval $0 \leq x \leq 1$ with help of the *hit-or-miss method*.

a) Choose therefore $a = 0$, $b = 1$, $h = 1$, $f(x) = \sqrt{1 - x^2}$ and multiply the result F_n with 4. Determine F_n as a function of n (i.e. for different values of n) and plot the difference of the exact result and F_n into a double-log diagram over $\log n$. (2 P)

b) Also use the *sample-mean method* for determination of F_n with $n \geq 10^4$. How large must n be chosen (so, how man trials are required) to get two accurate decimals? How does the error in F_n decrease with increasing n? (2 P)

4. Task *Random walk* (3 P)
Complete the intermediate steps in the given example for photons in the Sun, so:

a) Determine the mean free path length, assume that the Sun consists only of ionized hydrogen. (1 P)

b) Apply the random walk model to verify the formula for the time estimate. (2 P)