Exercise 11

C/C++ Numerical Integration, Monte Carlo simulations

(25.06.2025, hand in 02.07.2025)

1. **Task** *Numerical integration of a time series* (5 P)
   Integrate the "measured" count rate $\dot{C}(t)$

$$\frac{dC(t)}{dt} = e^{-t} \quad \rightarrow \quad C(1) = \int_0^1 e^{-t} dt \tag{1}$$

   with

   a) the Trapezoid Rule (2 P)

   b) Simpson's Rule (2 P)

   Compute the relative error $\epsilon = (C_{\text{exact}} - C_{\text{approx}})/C_{\text{exact}}$ for $N = 2, 10, 20, 100, 1000$ and plot the relations $\epsilon(N)$ for a) and b) in a log-log diagram. Which relations can be found? (1 P)

2. **Task** *Numerical derivatives* (4 P)
   Write a C++ program (using double precision) to apply the forward, central, and extrapolated difference algorithm to compute the *numerical derivative* of $\exp(x)$ for $x = 100$.

   a) Print out a table with the relative error $\mathcal{E}$ for each of the three methods in dependence on the step width h with decreasing h, until machine precision is reached (i.e. $h \approx \epsilon_{\text{m}}$). (2 P)

   b) Make a plot of $\log_{10} |\mathcal{E}|$ over $\log_{10} h$ and check the dependence of the *approximation error* on h as shown in the lecture, i.e. determine the slope. (2 P)

3. **Task** *Elastic neutron scattering* (11 P + 3 Bonus P)
   Write a program that simulates the neutron transport with the MC method shown in the lecture. (3 P)

   a) Structure your program as usual into a main program and (void) functions `initial()`, `scatter()`, `output()`, where in `main` a loop runs over all neutrons. Use a corresponding `makefile`! (1 + 1 P)

   b) Determine the fraction f_{c} of the captured (absorbed) neutrons, the fraction f_{r} of the reflected neutrons and the fraction f_{t} of the transmitted neutrons for $t = 1$, $p_{\text{c}} = p_{\text{s}}/2$ and following values of λ_{mfp}: 0.01, 0.05, 0.1, 1, 10. Start with 100 neutrons and increase that number until reproducable results are achieved.
   Explain the results qualitatively! (2 P)

   c) *Bonus:* Plot the paths of the individual neutrons with help of `Xgraphics` in the function `output` in the xy plane. How can one compute the y coordinate? (3 BP)

   d) Repeat the experiment with $t = 1$, $p_{\text{c}} = p_{\text{s}}$, $\lambda = 0.05$. Compare the result with the corresponding experiment from task (3b) and explain. (2 P)

   e) Repeat the task (3d) with $t = 2$ and $\lambda = 0.1$. Answer before the experiment: Does f_{t} and so on depend each on the individual values of t and λ, or is only the ratio λ/t crucial? (2 P)