Exercise 10
C/C++ Root finding, interpolation
(18.06.2025, hand in 25.06.2025)

---

1. Task *Interpolation* (8 P)

Try to interpolate the given cross section data from the lecture:

a) globally, by a Langrange polynomial (4 P)

b) piecewise, by Cubic Hermite spline interpolation (4 P).

Plot the given data points and the interpolated curve (points with sufficiently fine spacing) in a diagram.

2. Task *Root finding* (4 P)

Find solutions, i.e. energies E_B for the quantum states in a square well

$$\sqrt{V_0 - E_\mathrm{B}} \tan\left(\sqrt{V_0 - E_\mathrm{B}}\right) = \sqrt{E_\mathrm{B}} \tag{1}$$

with help of

a) the bisection method

b) the Newton-Raphson method

Set $V = 50$ and plot also Eq. (1) to get an idea for the start values. It may difficult to find all roots (especially with the Newton-Raphson method), so you may rather use an alternative form of Eq. (1)

$$\sqrt{E_\mathrm{B}} \cot\left(\sqrt{V - E_\mathrm{B}}\right) - \sqrt{V - E_\mathrm{B}} = 0 \tag{2}$$

3. Task *Kepler's equation* (4 P)

Remember Kepler's equation $E(t) - e \sin E(t) = M(t)$ as a tool to compute the position (eccentric anomaly E) of some celestial body, e.g., Jupiter (semimajor axis $a = 5.204491\,\mathrm{AU}$ and excentricity $e = 0.047837$).

Determine the *true anomaly* ϕ and *distance to the Sun* of Jupiter for a mean anomaly of $M = 199\overset{\circ}{.}7855$ (=JD 2 445 053.5). For the computation of the eccentric anomaly E use:

a) fixed-point iteration (see corresponding slides ...)

b) Newton's method

to get an accuracy of $0\overset{\circ}{.}00001$. Which method converges faster? Check the convergence "speed" also with a higher eccentricity, e.g., $e = 0.1$.

Note that M is given in degree!