Exercise 8

C/C++: The special three-body problem – Runge-Kutta method

(28.05.2025, hand in: 04.06.2025)

1. **Task** *The special three-body problem* (7 P)
   Implement the equations of motion which are mentioned in the lecture for the case of the special three-body problem with help of the *Euler-Richardson method* as an extension to the two-body problem.

   a) Both planets should in principle correspond to Earth and Jupiter, but for better demonstration we choose a ratio for the planetary masses of:

   $$m_1/M = 10^{-3} \quad \& \quad m_2/M = 4 \times 10^{-2} \tag{1}$$

   so that,

   ```
   ratio[0] = 0.04 * GM
   ratio[1] = -0.001 * GM
   ```

   Moreover, we start with circular orbits at

   2.52 AU and 5.24 AU.

   Use arrays for clarity

   ```
   float pos[np][nd], vel[np][nd] ;
   ```

   where `np = 2` is the number of planets and `nd = 2` are the x- and y-components. (4 P)

   b) Both planets start on circular orbits (initial value for v_y). What do the trajectories look like? Which planet is more influenced? Which trajectory is obtained, if the program uses more realistic planetary masses and distances (e.g., Earth and Jupiter, or Jupiter and Saturn)? (2 P)

   c) If you extend the program to the case of `np = 3`, how does the number of necessary calculations steps (calculation of the distances) scales with `np`? (1 P)

2. **Task** *Runge-Kutta method (RK4)* (11 P)
   Modify your original program `planet` so that the *fourth-order Runge-Kutta method (RK4)* instead of the Euler method the is used (4 P).

   a) As a measure for the accuracy of the different integration methods use the *relative* change in energy $\Delta E/E_{\text{ini}}$ after 100 orbits. Which method (explicit Euler, Euler-Cromer, Euler-Richardson, RK4) is more accurate for the same step size? Create one appropriate figure, i.e. with the measure of accuracy over time step size for the different methods in a log-log representation. Also compare the accuracy of RK4 for use with `float` variables instead of `double` variables. (5 P)

   b) One problem in N-body simulations with fixed (time) step size are close encounters, as they occur, e.g., for elliptical orbits of high eccentricity at perihelion. Is a Runge-Kutta method more appropriate for such problems? (2 P)