FULL TITLE
ASP Conference Series, Vol. **VOLUME**, **YEAR OF PUBLICATION**
NAMES OF EDITORS

Time-dependent simulation of a multicomponent stellar wind

V. Votruba[1,3], A. Feldmeier[2], J. Kubát[1], R. Nikutta[2]

[1] *Astronomický ústav, Akademie věd České republiky, 251 65 Ondřejov, Czech Republic*
[2] *Astrophysik, Institut fur Physik, Universitat Potsdam, Am Neuen Palais 10,14469 Potsdam, Germany*
[3] *Ústav teoretické fyziky a astrofyziky, Přírodovědecká fakulta, Masarykova univerzita, Kotlářská 2, 611 37 Brno, Czech Republic*

Abstract. We develop a time-dependent multicomponent code for stellar wind from hot stars and apply them to some typical Be stars. We calculate the Chandrasekhar dynamical friction using simple a approximation of the Chandrasekhar function, which help us stabilize the stiffness of the problem. We use the vanLeer code to solve time dependent hydrodynamics equations of a two component stellar wind. We represent Coulombic collisions between the passive bulk of plasma and the absorbing ions by calculating approximations of the Chandrasekhar function, which allow us to use an analytical expression for the friction term in the code. We apply this method for stars with well coupled wind in the first and compare result with usual one component mCAK model.

1. Introduction

The idea that the stellar material is accelerated by absorbtion and scattering of the star's radiation was originally introduced by Milne (1926). This idea was developed in full theory by Castor, Abbott, & Klein (1975), later referred to as the CAK theory. In this theory, the stellar wind is described by a steady spherically symmetric one-component flow from a nonrotating star, and the radiative acceleration, which is acting on a whole plasma, is given mainly by the interaction of ultraviolet photospheric radiation with resonance lines of ions such as C IV, N V, and Si IV. The radiative force is parameterized by CAK constants k, α. Moreover, they assumed point source approximation, which means that all radiation was assumed to come from a point at the center of the star. Although qualitative agreement between theory and observations was achieved, there were still some discrepancies remaining, namely the terminal velocity v_∞ being too low,and the mass-loss rate $\dot{M}$ being too high. Further development of theory was desirable to precise and improve predictions. Abbott (1982) added another parameter to the theory, namely the dependence of the radiative force on the ionization state of the stellar wind, which lead to a three parameter description of the driving force using the parameters α, k, δ. Moreover, he included a complete list of atomic data spanning from the first to the sixth ionization stage of the elements H-Zn to calculate the radiative acceleration and obtained more precise values of the CAK constant. The point star size approximation lead to an underestimate of the radiative force near the photosphere. Another major stride was made by Pauldrach, Puls, & Kudritzki (1986), where the authors abandoned this assumption, and modified the CAK theory (referred to as the mCAK theory) by using radiative force with correction for a finite size of a disk.

The approximation of one-component flow used by the CAK theory is acceptable for most cases of stellar winds from O stars and some B stars. But in reality the radiation is acting on absorbing ions and electrons only, and those particles share momentum through Coulombic collisions with the remaining passive part of the plasma (namely protons). The Dynamical effect of the Coulombic collisions on the plasma is well described by dynamical friction, which was first used by Chandrasekhar (1943) for the case of the gravity force, and later it was used for the electromagnetic force by Spitzer (1956). As was first shown by Springmann & Pauldrach (1992), this more detailed multicomponent description of stellar wind predicts a runaway mechanism. It means that under certain conditions, namely low density of the wind, coulombic interactions are so small that they stop the momentum transfer between the passive bulk of plasma and absorbing ions and as a result, wind decouples at a certain point. From this point, called the decoupling radius, absorbing ions are highly accelerated while passive plasma is decelerated. Moreover with the help of some approximations they derived an analytical formula for the decoupling radius. Later, Krtička & Kubát (2000) obtained a contradictory result from numerical calculations. They found a nondecoupling solution with slow acceleration, which is analogous to a shallow solution from the mCAK theory with a low density wind. This contradictory result was investigated by Owocki & Puls (2002) by means of a linear stability analysis of the multicomponent wind. They predicted that flows should be disrupted by ion separation before reaching a solution with slow acceleration.

A very interesting result is the generation of pulsating shells, which was first discovered by Porter & Skouza (1999). In the case when winds decouple at the point where the local velocity of the flow is still smaller than the escape velocity, passive plasma is still gravitationally bound to the star. It implies that the matter is decelerated and falls, back down to the star. Interaction with an outflow from the star leads to the pulsating shells. Unfortunatelly this result is based on the one-component model only, with an artificial turn off of radiative acceleration, and not on the multicomponent description of the wind.

2. Multicomponent description of the stellar wind

We start with time-dependent forms of relevant hydrodynamics equations for a multicomponent radiatively driven flow. We restrict ourselves to standard assumptions of 1D spherically symmetric outflow and to two components only, namely absorbing ions and passive plasma (protons). We also neglect the effect of macroscopic magnetic and electric fields and assume plasma quasineutrality. Acting forces on absorbing ions are gravity, dynamical friction, pressure gradient, and radiation, while on passive plasma they are only gravity, pressure gradient, and dynamical friction. Continuity equations for both components are of the form (p stands for passive plasma and i stands for absorbing ions)

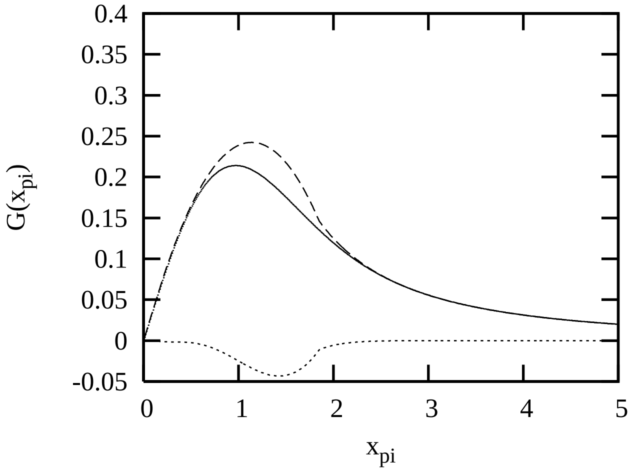

Figure 1. The plot of the Chandrasekhar function. The exact function $G(x)$ according to (11) is plotted by a solid line, the approximation $G_A(x)$ after (16) by a dashed line, and the diference between both $G(x) - G_A(x)$ by a dotted line.

$$\frac{\partial \rho_p}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho_p v_p)}{\partial r} = 0\,, \tag{1a}$$

$$\frac{\partial \rho_i}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho_i v_i)}{\partial r} = 0\,. \tag{1b}$$

Similarly, the equations of motion

$$\frac{\partial v_p}{\partial t} + v_p \frac{\partial v_p}{\partial r} + + \frac{1}{\rho_p}\frac{\partial p_p}{\partial r} = \frac{R_{pi}}{\rho_p} - g_*\,, \tag{2a}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial r} + + \frac{1}{\rho_i}\frac{\partial p_i}{\partial r} = g_{\mathrm{rad}} - g_* - \frac{R_{pi}}{\rho_i}\,, \tag{2b}$$

Here ρ_i, v_i, p_i are the density, velocity, and the pressure of the ions. Similarly, ρ_p, v_p, p_p are the density, velocity, and pressure of the passive plasma. Further, g_* is the effective gravitational acceleration, and R_{pi} is the frictional force, which is described in the text below (see Eq. 8). As another major approximation we use the assumption of isothermality. The validity of these approximations is discussed later in the text. Then we include state equations for partial gas pressures for both components to the set of equations,

$$p_p = a_p^2 \rho_p \tag{3a}$$

$$p_i = a_i^2 \rho_i. \tag{3b}$$

We take radiative acceleration in the form

$$g_{\mathrm{rad}}(r) = \frac{\eta^{\alpha-1}\left(\sigma_e^{\mathrm{ref}}\right)^{1-\alpha}}{4\pi {v_{th}}^{\alpha}} \frac{L_*}{r^2} k \left(\frac{1}{\rho_i}\frac{\partial v_i}{\partial r}\right)^{\alpha} f_{\mathrm{ion}} f_{\mathrm{fin}} \tag{4}$$

with mCAK force multipliers k, α, δ. Here L_* is the luminosity of the star, $v_{\rm th}$ is the thermal velocity of the ions, and $\sigma_e^{\rm ref}$ is a reference value of opacity for Thomson scattering. Because now radiative acceleration is acting on absorbing ions only, we used a scalling factor η with the value $\eta = 0.0127$ for solar abundance. Finally, $f_{\rm fin}$ is the correction for a finite size disk of the star (Castor et al. 1975; Friend & Abbott 1986; Pauldrach et al. 1986)

$$f_{\rm fin}(r) = \frac{(1+\sigma)^{\alpha+1} - (1+\sigma\mu_*^2)}{\sigma(\sigma+1)^\alpha(1-\mu_*^2)(\alpha+1)}\,, \tag{5}$$

where σ is given by

$$\sigma = \frac{r}{v_i}\frac{\partial v_i}{\partial r} - 1 \tag{6}$$

and $f_{\rm ion}$ is the correction for an ionization state of the stellar wind,

$$f_{\rm ion}(r) = \left(\frac{10^{-11}{\rm cm}^3\, n_e}{W(r)}\right)^\delta, \tag{7}$$

where $W(r)$ corresponds to the geometric dilution factor. Number density of the electrons n_e roughly corresponds to the number density of passive plasma. Interaction between passive plasma and absorbing ions is provided by Coulomb collisions. Those interactions are described by the frictional force R_{pi} per unit volume, using the well known Chandrasekhar formula for dynamical friction (see Chandrasekhar 1943; Spitzer 1956)

$$R_{pi} = n_p n_i k_{pi} G(x_{pi}) \tag{8}$$

Here $n_p\,, n_i$ are number densities of passive plasma and absorbing ions, respectively, the frictional coefficient $k_{\rm pi}$ is given by

$$k_{pi} = \frac{4\pi \ln\Lambda {Z_p}^2 {Z_i}^2 e^4}{k_B T}\frac{v_i - v_p}{|v_i - v_p|}, \tag{9}$$

where $Z_i\, e$,$Z_p\, e$ are the ion and passive plasma charges, respectively, and $\ln\Lambda$ is the Coulomb logarithm, defined as

$$\ln\Lambda = \ln\left[\frac{24\pi}{\sqrt{n}}\left(\frac{kT}{4\pi e^2}\right)^{1.5}\right], \tag{10}$$

k_B is the Boltzmann constant, n is the total number density, T is the wind temperature, and $G(x)$ is the Chandrasekhar function which is defined in terms of the error function $\Phi(x_{pi})$ Spitzer (1956)

$$G(x_{pi}) = \frac{\Phi(x_{pi})}{2x_{pi}^2} - \frac{\exp\left(-x_{pi}^2\right)}{2\sqrt{\pi}} \tag{11}$$

This function depends on the ion separation drift speed relative to the passive plasma x_{pi}, scaled to the mass-weighted thermal speed (Owocki & Puls 2002)

$$x_{pi} = \frac{|v_i - v_p|}{v_{th}\sqrt{1 + A_i/A_p}}, \tag{12}$$

A_i and A_p stand for the mean atomic mass of ions and passive plasma in atomic units.

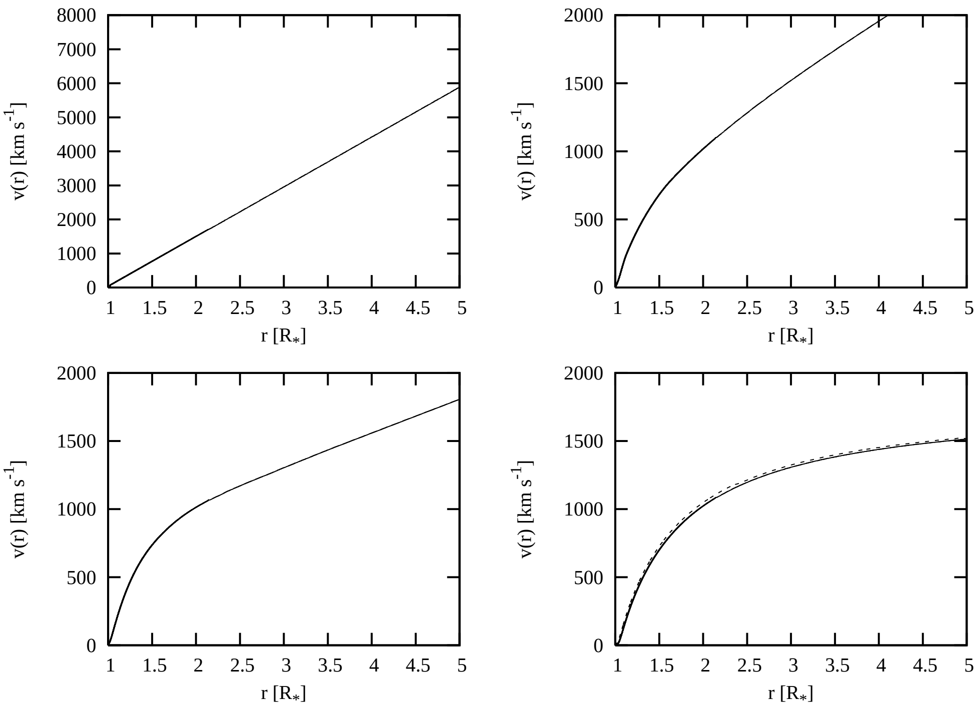

Figure 2. *Upper left panel:* The initial condition of velocity for a time dependent two-component model of a τ Sco star from Table 1. This star was chosen due to its predicted well coupled stellar wind. *Upper right panel:* After few time steps we can see, how the velocity profile is attracted to the CAK solution. *Lower left panel:* The same situation but much later and closer to the CAK solution. *Lower right panel:* Finally, it converged to the CAK type solution. We compared our two-component model with a one-component simulation. We see that the two-component solution (dashed line) corresponds with the one component solution (solid line).

3. Method of solution

For solving a coupled system of hydrodynamical equations we used the classical Eulerian scheme. Equations (1) and (2) are discretized using an operator-splitting time-explicit finite differences method on a staggered mesh. Advection fluxes are calculated using van Leer's monotonic interpolation see (Van Leer 1982). To satisfy the Courant condition of stability we use a time step, which is the smallest value from calculated time steps from the Courant conditions for both components, absorbing ions and passive plasma. As an initial condition we used an exponential velocity profile for the subsonic part and a linear velocity profile for the supersonic part. Initial density for both components is then calculated from the equations of continuity. To calculate the radiative force we used tabulated values of the CAK constants from (Abbott 1982). The most important problem which has to be solved is the inclusion of the friction term. If this term is included, it leads to a big stiffness of equations. Roughly speaking, we are attempting to model stellar wind on a relatively slow time scale, which represents timescale for radiative processes in a situation where the faster processes, which correspond to friction processes maintain local equilibrium. Due to this fact, we approximate the friction term by an analytical expression. Namely, we approximate the Chandrasekhar function by the formula which is analytically integrable. We use three different aproximations applicable to three parts of the drift speed interval. For small drift velocity we use (Owocki & Puls 2002)

$$G_1(x_{pi}) \approx \frac{2x_{pi}}{3\sqrt{\pi}} \quad \text{for}\, x \leq x_{12} \tag{13}$$

for large drift speed we use (Owocki & Puls 2002)

$$G_3(x_{pi}) \approx \frac{1}{2{x_{pi}}^2} \quad \text{for}\, x \geq x_{23} \tag{14}$$

and finally for values in between we interpolate using a quadratic function,

$$G_2(x_{pi}) \approx ax_{pi}^2 + bx_{pi} + c \tag{15}$$

where parameters a, b, c are chosen to achieve the best fit. For our model we use $a = -0.176$, $b = 0.404$, $c = -0.0011$, and $x_{12} = 0.15$, $x_{23} = 1.85$. The whole aproximation is given by the sum

$$G_A(x_{pi}) = G_1(x_{pi}) + G_2(x_{pi}) + G_3(x_{pi}) \tag{16}$$

The function $G_A(x)$ and the difference from true Chandrasekhar function is shown in Fig 1. We can see that in the dynamicaly most important part $x_{pi} \leq x_{pi}^{\max} = 0.24$ of the Chandrasekhar function the difference is very small. Using Eqn. (16) we are able to calculate the friction term from the analytical expression at every time step. Boundary conditions were set according to characteristics.

Table 1. Model parameters

Star	M [$M_\odot$]	$T_{\rm eff}$ [K]	R [$R_\odot$]	q_i/q_p	α	k	δ	v_∞ [km.s^{-1}]
B0	90.0	28 500	37	3.0	0.590	0.170	0.09	1800
κ Cas	29	21600	35.4	3.0	0.5	0.287	0.089	800
τ Sco	20	32000	6.7	3.0	0.609	0.156	0.057	1600

For absorbing ions we used a fixed density and an extrapolated momentum flux at the inner boundary and both extrapolated at the outer boundary. For passive plasma both variables are fixed at the inner boundary and extrapolated at the outer boundary. As will be discused in the next section, problems arise when decoupling occurs, because it changes the outgoing and incoming characteristic configuration.

4. First simulation

As the first test, we use our code for stars where a well coupled wind is predicted. More specifically, we test K Cas, τ Sco star and some a $B0$ star. We used the basic stellar parameters for κ Cas and τ Sco we found in Wilson & Dopita (1985). Stellar parameter for $B0$ star were taken from Krtička & Kubát (2000). We summarize our model parameters in Tab. 1. We compare the result of the simulation with a one-component simulation of the star. As we expected, after few time steps the solution converged to a mCAK solution, with a corresponding velocity, v_∞. Computed velocity v_∞ is also in Tab. 1. This process is illustrate in Fig. 2.

5. Conclusions

For a well-coupled stellar wind we see that after a few time steps, results converge from arbitrary initial conditions to a solution which corresponds with the CAK solution. We checked our code on a set of different B stars. Our next step will be to prove the influence of our assumptions, namely the assumption of isothermality. This assumption isn't strictly valid, due to the presence of frictional heating in multicomponent stellar winds. We will also analyse the effect of decoupling on our method and how this dynamically important effect will influence our simulations. Also we will test result from Porter & Skouza (1999) about periodic shell around the star.

Acknowledgments. This work was supported by a grant of the Grant Agency of the Academy of Science of the Czech Republic B301630501 and by project AV0Z10030501.

References

Abbott, D. C. 1982, ApJ 259, 282
Friend, D. B., Abbott, D. C. 1986, ApJ 311, 701
Castor, J. I., Abbott D. C., & Klein, R. I. 1975, ApJ 195, 879
Chandrasekhar, S. 1943, Principles of Stellar Dynamics, University of Chicago Press
Krtička, J., Kubát J. 2000, A&A 359, 983
Milne, E. A. 1926, MNRAS 86, 459
Owocki, S. P., Puls, J. 2002, ApJ 568, 965
Pauldrach, A., Puls, J., Kudritzki, R. P. 1986, A&A 164, 86
Porter, J. M., Skouza, B. A. 1999, A&A 344, 205
Springmann, U. W. E., Pauldrach, A. W. A. 1992, A&A 262, 515
Spitzer, L. 1956, Physics of fully ionized gases, New York Interscience
Van Leer, B., Proc. 8th International Conference on Numerical Methods in Fluid ynamics, Berlin:Springer Verlag, 1982
Wilson I.R.G., Dopita M.A., 1985, A&A 149, 295

Discussion

S. Owocki: I am glad to see you pursue this challenging problem. Two concerns regards the stiffnes of the coupled equations in dense limit, and the fast nature of the ion Abbott mode in the separation limit. I'ts not clear how one bridges these two limits. Moreover in the separation limit, the high speed of ion Abbott could seem effectively to make the whole outflow subcritical to this wave mode. This raises the issue of the proper formulation of the outer boundary, since you

can no longer simply assume that all the wave characteristic are outward pointing there.
Ph. Stee: Why have you used a so high mass and large stellar radius for you simulation ? It seem more to be parameters for a LBV than a B0 star.
V.Votruba: I used B0 star and stellar parameters from Krtička & Kubát (2000), because we want to compare result from stationary and time dependent code. But of course, we studied more than only one star, as we can see from Tab. 1.