Exercise 6
C/C++ Runge-Kutta method – The special three-body problem
(28.05.2020, hand in: 04.06.2020)

1. Task Runge-Kutta method (RK4) (11 P)
Modify your original program planet so that instead of the Euler method the fourth-order Runge-Kutta method (RK4) is used (4 P).

   a) Which method is more accurate for the same step size and how could one measure that? Also compare these methods for use with double variables instead of float variables. Create one appropriate figure, i.e. with the measure of accuracy over time step size for the different methods. (5 P)

   b) One problem in N-body simulations with fixed (time) step size are close encounters, as they occur, e.g., for elliptical orbits of high eccentricity at perihelion. Is a Runge-Kutta method more appropriate for such problems? (2 P)

2. Task The special three-body problem (7 extra P)
Implement the equations of motion which are mentioned in the lecture for the case of the special three-body problem with help of the Euler-Richardson method as an extension to the two-body problem.

   a) Both planets should in principle correspond to Earth and Jupiter, but for better demonstration we choose a ratio for the planetary masses of:

\[
m_1/M = 10^{-3} \ & \ m_2/M = 4 \times 10^{-2}
\]

so that,
\[
\text{ratio}[0] = 0.04 * \text{GM} \\
\text{ratio}[1] = -0.001 * \text{GM}
\]

Moreover, we start with circular orbits at 2.52 AU and 5.24 AU.

Use arrays for clarity

\[
\text{float pos[np][nd], vel[np][nd]};
\]

where np = 2 is the number of planets and nd = 2 are the x- and y-components. (4 extra P)

   b) Both planets start on circular orbits (initial value for \(v_y\)). What do the trajectories look like? Which planet is more influenced? Which trajectory is obtained, if the program uses more realistic planetary masses and distances (e.g., Earth and Jupiter, or Jupiter and Saturn)? (2 extra P)

   c) If you extend the program to the case of np = 3, how does the number of necessary calculations steps (calculation of the distances) scales with np? (1 extra P)