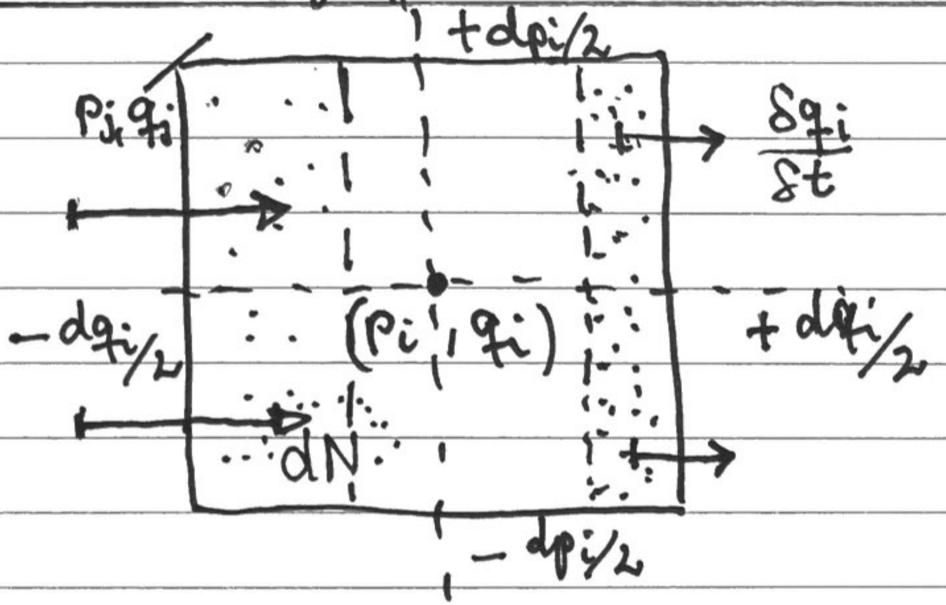
Herleitung „Ausfluss" bei Satz von Liouville : $dN = n\, dV$, $dV = dp_1 dq_1 \dots dp_n dq_n$



I) Phasenraum-kinematik

$$\Pi_i = dp_1 dq_1 \cdots \cancel{dp_i}\,\cancel{dq_i} \cdots dp_n dq_n$$

$$\left.\frac{\delta(dN)}{\delta t}\right|_i = \frac{\left[-n\left(q_i+\frac{dq_i}{2}\right)\frac{\delta q_i}{\delta t}\left(q_i+\frac{dq_i}{2}\right) + n\left(q_i-\frac{dq_i}{2}\right)\frac{\delta q_i}{\delta t}\left(q_i-\frac{dq_i}{2}\right)\right] dp_i\, dq_i\, \Pi_i}{dq_i}$$

$$+ \frac{\left[-n\left(p_i+\frac{dp_i}{2}\right)\frac{\delta p_i}{\delta t}\left(p_i+\frac{dp_i}{2}\right) + n\left(p_i-\frac{dp_i}{2}\right)\frac{\delta p_i}{\delta t}\left(p_i-\frac{dp_i}{2}\right)\right] \underbrace{dq_i\, dp_i\, \Pi_i}}{dp_i}$$

(hier steht $\frac{f(x+dx/2)\,g(x+dx/2) - f(x-dx/2)\,g(x-dx/2)}{dx} = (fg)'(x)$)^{dV}

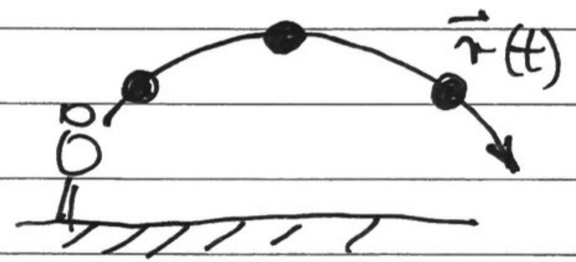
$$0 = \frac{\partial w}{\partial t} + \frac{\partial (w \dot{q}_i)}{\partial q_i} + \frac{\partial (w \dot{p}_i)}{\partial p_i}$$

Summenkonvention

$$= \frac{\partial w}{\partial t} + \frac{\partial w}{\partial q_i} \dot{q}_i + \frac{\partial w}{\partial p_i} \dot{p}_i + w \frac{\partial \dot{q}_i}{\partial q_i} + w \frac{\partial \dot{p}_i}{\partial p_i}$$

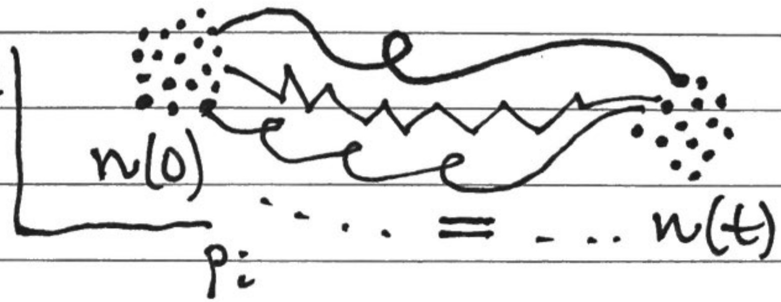
$$= \frac{\partial w}{\partial t} + \frac{\partial w}{\partial q_i} \dot{q}_i + \frac{\partial w}{\partial p_i} \dot{p}_i + w \cancel{\left(\frac{\partial}{\partial q_i} \frac{\partial H}{\partial p_i} - \frac{\partial}{\partial p_i} \frac{\partial H}{\partial q_i} \right)}$$

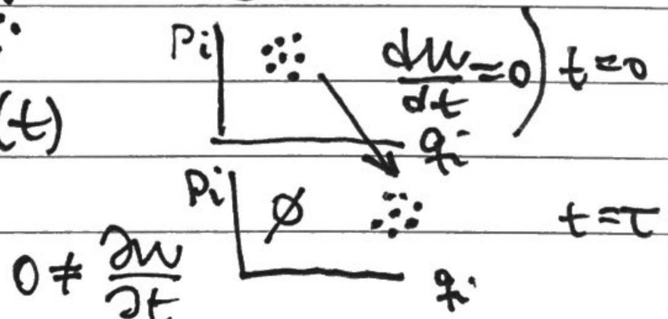
$$\left[\begin{array}{l} w = w(p_1, \ldots, p_N, q_1, \ldots, q_N, t) \\ \text{d.h. } \dfrac{dw}{dt} = \dfrac{\partial w}{\partial t} + \dfrac{\partial w}{\partial p_i} \dot{p}_i + \dfrac{\partial w}{\partial q_i} \dot{q}_i \end{array} \right]$$



Newton $\frac{d}{dt}$ von Ball

$$= \boxed{\frac{dw}{dt} = 0}$$





Satz von Liouville

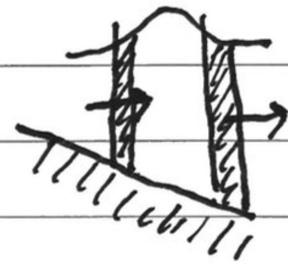
$$\frac{\partial}{\partial t}\underbrace{\left(\frac{dN}{dV}\right)}_{n}\Bigg|_i = -\frac{\partial}{\partial q_i}(n\dot{q}_i) - \frac{\partial}{\partial p_i}(n\dot{p}_i)$$

$$\boxed{\frac{\partial}{\partial t}\, n + \sum_{i=1}^{n}\left[\frac{\partial}{\partial q_i}(n\dot{q}_i) + \frac{\partial}{\partial p_i}(n\dot{p}_i)\right] = 0}$$ Kontinuitäts-gleichung

vgl Hydro Lamb $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

Kontinuitätsglg.

Klausurvorbereitung: Kontin. glg. für Wasserwellen



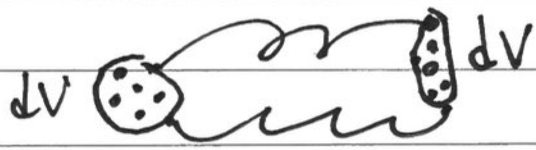
siehe auch Herleitung der Divergenz

II) Hamilton

$$n = \frac{dN}{dV}$$

$$0 = \frac{dn}{dt} = \frac{d}{dt}\left(\frac{dN}{dV}\right)$$

$\frac{d}{dt}(dN) = 0$ weiß ich $\frac{d}{dt}N = 0$



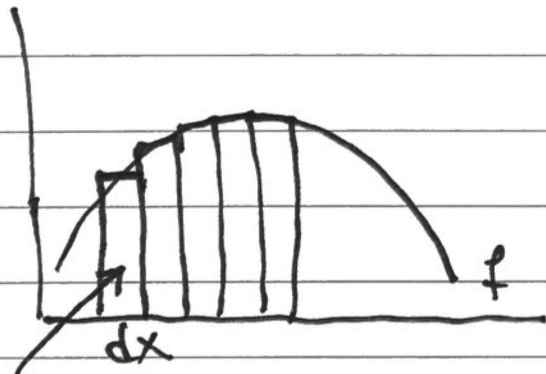
also $\boxed{\frac{d}{dt}\,dV = 0}$

Satz von Liouville (Form. II)

$V = \int dV$ also

$$\boxed{\frac{dV}{dt} = 0} \quad \text{(Form. III)}$$

vgl. Riemann-Summe des Integrals



infinit. Fläche unter f

$\approx f_{fix}\,dx$

$\int df = f$ ohne Fehler!

Liouville: das Phasenraumvolumen ist ~~zeitl~~ zeitlich konstant

„area-preserving maps"