W-W-V: φ, J

$H(p,q) \overset{?}{\rightarrow} K(J)$

$\dot\varphi = \frac{\partial K}{\partial J}$, $\quad \dot J = -\frac{\partial K}{\partial \varphi} \equiv 0$

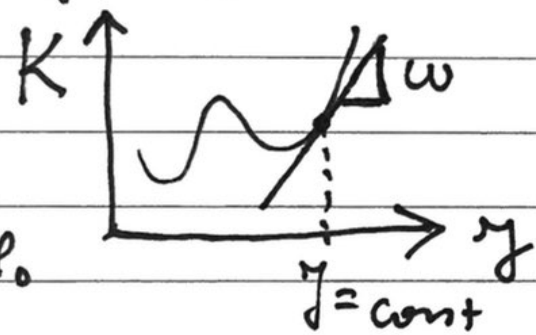
$J = \text{const}$

$= \omega$

$= \text{const}$

also

$\varphi(t) = \omega t + \varphi_0$



damit Lsg des mech. Probl.

$J_i = \text{const}_i \quad (i = 1, \dots, n)$

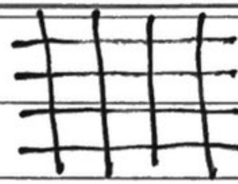
$\varphi_i = \omega_i t + \varphi_{i0} \quad (i = 1, \dots, n)$

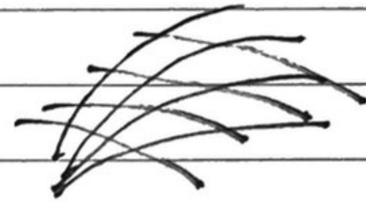
Liouville

dabei stillschweigend angenommen dass alle φ_i unabh. sind.

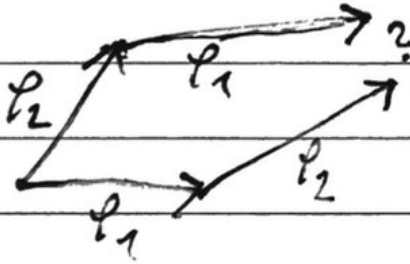
Liouville: J_i müssen in Involution sein

z.B. Kart. Koord.



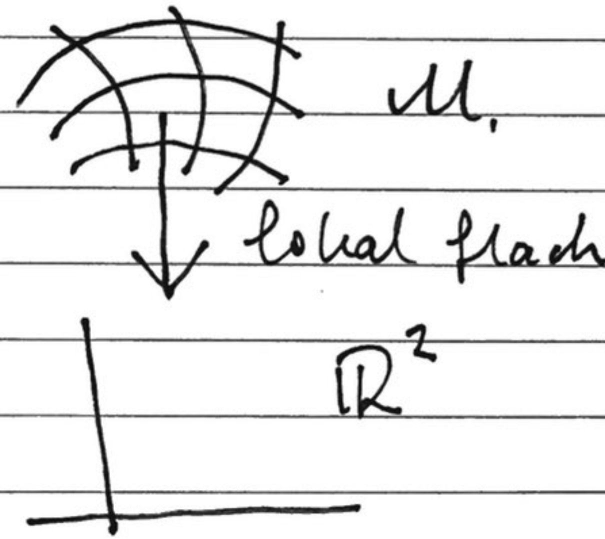


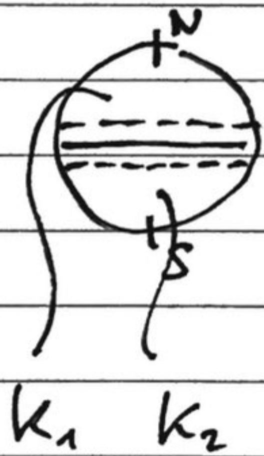
Könnte auch passieren

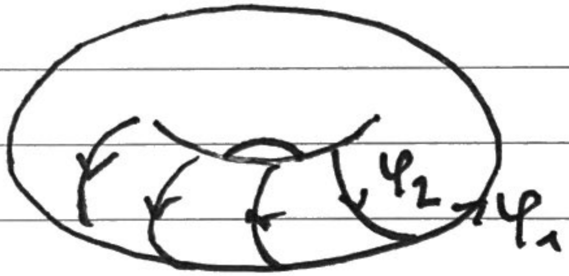


„Involution"

diffbare Mannigfaltigkeit







periodisch

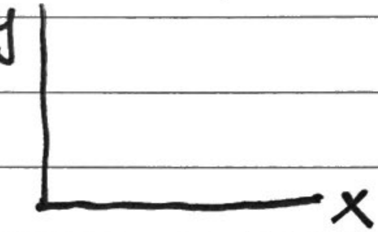
„quasi-periodisch"
Bahn füllt Torus

Von Neumann:

$\omega_1 : \omega_2 \in \mathbb{Q}$: periodisch

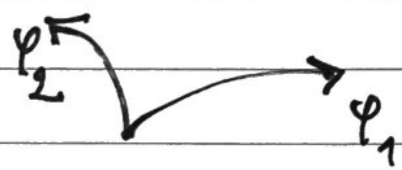
$\omega_1 : \omega_2 \in \mathbb{R}\setminus\mathbb{Q}$: quasi-periodisch

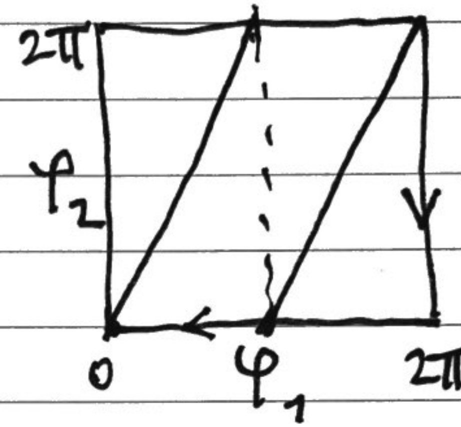
Kart.:



Mannigfaltigkeit

Leibniz

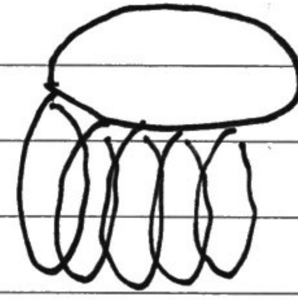


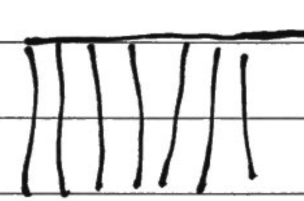


(schwarz)

$\omega_2 = 2\omega_1$ $\qquad$ $\omega_2 = \pi\omega_1$

warum Torus? $\underbrace{S_1 \times \dots \times S_1}_{n} = T_n$





Beispiele W-W-V

1) Oszillator ... wie immer

2) Schwerkraft

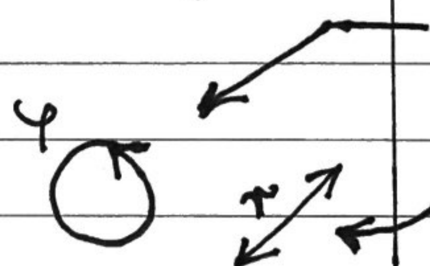
$$T = \frac{1}{2} m \left(\dot r^2 + r^2 \dot\varphi^2\right)$$

$$p_r = \frac{\partial \mathcal{L}}{\partial \dot r} = \frac{\partial T}{\partial \dot r} = m \dot r$$

$$p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot\varphi} = \frac{\partial T}{\partial \dot\varphi} = m r^2 \dot\varphi$$

$$\overset{I_2}{\overset{\shortparallel}{H}} = \underbrace{\frac{1}{2m}\left(p_r^2 + \frac{\overset{I_1}{\overset{\shortparallel}{p_\varphi}}{}^2}{r^2}\right)}_{T} \underbrace{- \frac{\gamma m}{r}}_{V}$$

2 Freiheitsgrade

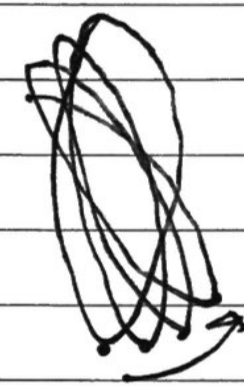


eigentlich sollte doch Keplerbahn so sein

Rosette

Epizykel

Kreis auf Kreis



Merkur

Periheldrehung

W-W sind (Poincaré, Sommerfeld)

$$J_\varphi = \oint d\varphi \, \overset{\text{const}}{\overset{\shortparallel}{p_\varphi}} = I_1 \oint d\varphi = 2\pi I_1$$

$$J_r = \oint dr \, p_r = \oint dr \left[2m I_2 + \frac{2\gamma m^2}{r} - I_1^2/r^2\right]^{1/2}$$

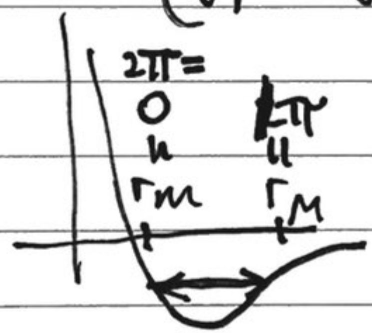
Ergebnis (S.F.)

$$J_r = -J_\varphi + \pi \gamma m \sqrt{\frac{2m}{-E}}$$

∇ ∘ Schreibe $E = H(J_r, J_\varphi)$

$$H(J_r, J_\varphi) = E = -\frac{2\pi^2 \gamma^2 m^3}{(J_r + J_\varphi)^2}$$

kein $\varphi_\varphi, \varphi_r$



damit

$$\omega_\varphi = \frac{\partial H}{\partial J_\varphi} = \frac{\partial H}{\partial J_r} = \omega_r \qquad \left(\begin{array}{l} \varphi_1 \equiv \varphi \\ \varphi_2 \equiv r \\ \quad \text{nur} \end{array} \right)$$