partielle Integration
≙ Integration by parts

δ-Distrib.

$$\int_{-\infty}^{\infty} dx\, \delta(x) = 1 \qquad f(x) = 0 \ \forall x \neq 0$$

$$\int_{-\infty}^{\infty} dx\, f(x)\, \delta(x) = f(0)$$

$$f(x_1)\,g(x_1) - f(x_0)\,g(x_0) = \int_{x_0}^{x_1} dx\, \Big[\underbrace{f(x)\,g(x)}_{F(x)}\Big]' = \int dx\, f'g + \int dx\, fg'$$

$$\int_{x_0}^{x_1} dx\, f'g = fg\Big|_{x_0}^{x_1} - \int_{x_0}^{x_1} dx\, fg'$$

was ist eine Variation?

Ein $\delta f(x) :=$

$$\varepsilon\, \eta(x)$$

mit $\varepsilon \ll 1$

& η beliebig

Euler ... puh ... $\eta = \delta$

Fundamentallemma der Variationsrechn.

$$\int_{x_a}^{x_e} dx\, f(x)\, g(x) = 0 \quad \text{für}$$

f & g stetig & g beliebig $\rightarrow f(x) = 0$

Herleitung E-L nach L

$$0 = \delta S = \delta \int dt \; \mathcal{L}(q, \dot{q})$$

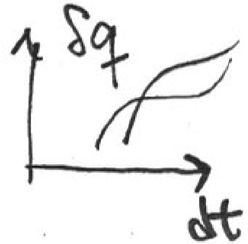
PAkW

$$= \int dt \; \delta \mathcal{L}(q, \dot{q})$$

$$= \int dt \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta \dot{q} \right]$$

$$= \int dt \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta \frac{d}{dt} q \right]$$

$$= \int dt \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot{q}}}_{f} \frac{d}{dt} \underbrace{\delta q}_{g} \right]$$



$$= \int_{t_A}^{t_B} dt \underbrace{\left[\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} \right]}_{F(t)} \delta q(t) + \cancel{\frac{\partial \mathcal{L}}{\partial \dot{q}} \delta q} \Big|_{t_A}^{t_B}$$

es sei $\delta q_A = \delta q_B =$

$$\boxed{\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} = 0}$$

E-L nach E

$$S = \dots + \mathcal{L}(q_0)\,dt + \dots$$

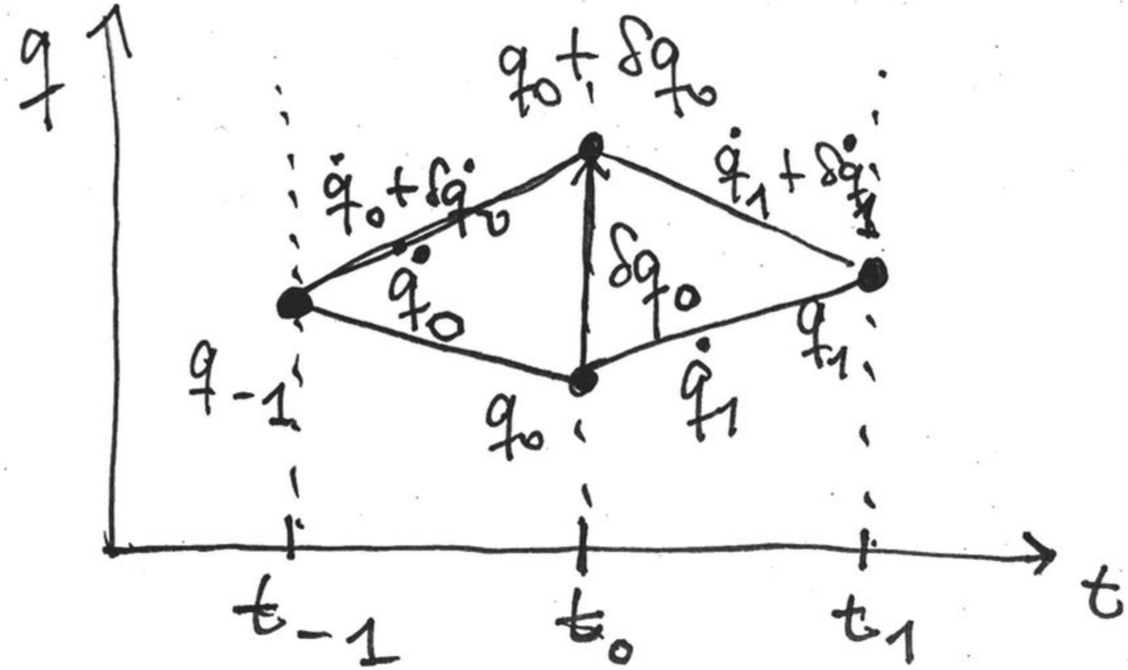
$$0 = \delta S = \mathcal{L}(q_0 + \delta q_0)\,dt - \mathcal{L}(q_0)\,dt \equiv \left.\frac{d\mathcal{L}}{dq}\right|_{q_0} dt \rightarrow \left.\frac{d\mathcal{L}}{dq}\right|_{q_0} = 0$$

$q(t_0)$

Jetzt aber $\mathcal{L}(q, \dot{q})$

Linksseit. Diff.

$$\dot{q}_0 = \frac{q_0 - q_{-1}}{dt}$$



$$\dot{q}_0 = \frac{q_0 - q_{-1}}{dt}$$

$$\dot{q}_1 = \frac{q_1 - q_0}{dt}$$

$$\dot{q}_0 + \delta \dot{q}_0 = \frac{q_0 + \delta q_0 - q_{-1}}{dt}$$

$$\dot{q}_1 + \delta \dot{q}_1 = \frac{q_1 - [q_0 + \delta q_0]}{dt}$$

$$\rightarrow \quad \delta \dot{q}_0 = \frac{\delta q_0}{dt}$$

$$\delta \dot{q}_1 = -\frac{\delta q_0}{dt}$$

$$\delta \frac{dq}{dt} = \frac{d}{dt} \delta q$$

$$0=\delta S = \dots + L(q_0+\delta q_0, \dot q_0 + \delta \dot q_0)\,dt \qquad (1)$$

$$+ L(q_1 + \cancel{\delta q_1}, \dot q_1 + \delta \dot q_1)\,dt + \dots \qquad (2)$$

Euler

$$-\Big[\dots + L(q_0, \dot q_0)\,dt \qquad (3)$$

$$+ L(q_1, \dot q_1)\,dt + \dots\Big] \qquad (4)$$

$$\cancel{\neq L(q_0+\delta q_0, \dot q_0 + \delta \dot q_0) = L(q_0+\delta q_0, \dot q_0)}$$

$$= \frac{\underset{1}{L\left(q_0 + \delta q_0, \dot q_0 + \delta \dot q_0\right)} - \underset{+N}{L\left(q_0 + \delta q_0, \dot q_0\right)}}{\delta \dot q_0} \, \delta \dot q_0 \, dt$$

$$+ \frac{\underset{2}{L\left(q_1, \dot q_1 + \delta \dot q_1\right)} - \underset{4}{L\left(q_1, \dot q_1\right)}}{\delta \dot q_1} \, \delta \dot q_1 \, dt$$

$$+ \frac{\underset{+N}{L\left(q_0 + \delta q_0, \dot q_0\right)} - \underset{3}{L\left(q_0, \dot q_0\right)}}{\delta q_0} \, \delta q_0 \, dt$$

$$= \frac{\partial L\left(q_0 + \delta q_0, \dot q_0 + \delta \dot q_0\right)}{\partial \dot q_0} \, \delta q_0$$

$$- \frac{\partial L\left(q_1, \dot q_1 + \delta \dot q_1\right)}{\partial \dot q_1} \, \delta q_0$$

$$+ \partial L\left(q_0 + \delta q_0, \dot q_0\right) / \partial q_0 \quad \delta q_0 \, dt$$