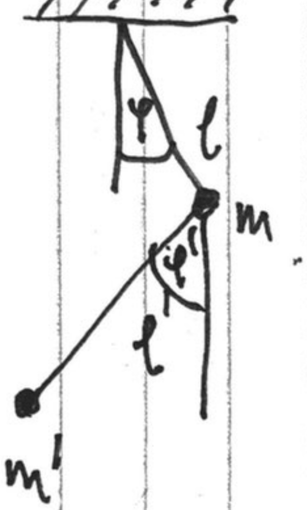
Verallgemeinerte Koordinaten – Variables



in Ebene:

$n = 2$ Freiheitsgrade

gesucht

$$0 = \delta \int dt\, \mathcal{L}\left(\varphi_1(t), \varphi_2(t), \dot{\varphi}_1(t), \dot{\varphi}_2(t)\right)$$

$$= \delta \int dx\, dy\, \mathcal{L}\left(z(x,y), z_x(x,y), z_y(x,y)\right)$$

$$z_y = \frac{\partial z}{\partial y}$$

Aufstellen der BGL:

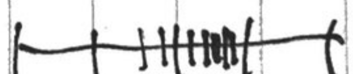
$$\mathcal{L}(\dots, \dots)$$

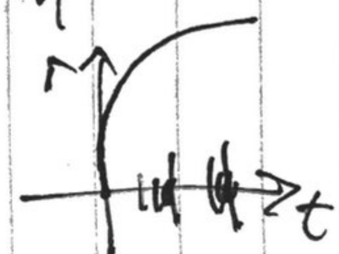
$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = 0$$

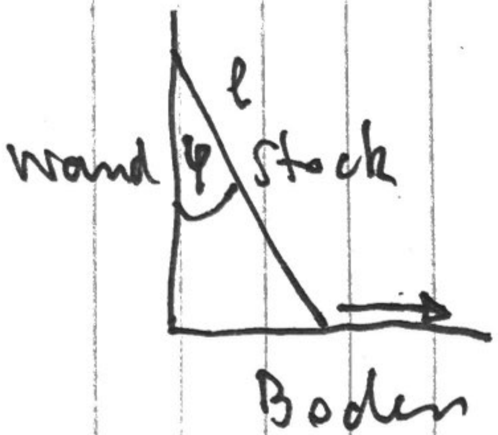
$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

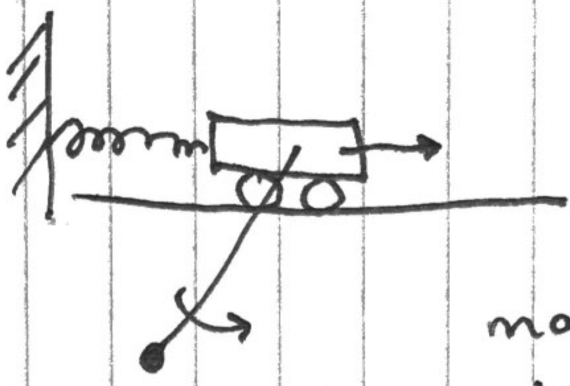
$q(t)$

$q(s(t))$



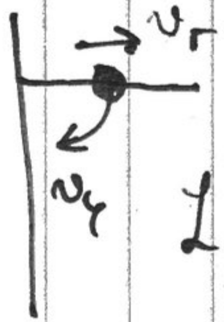






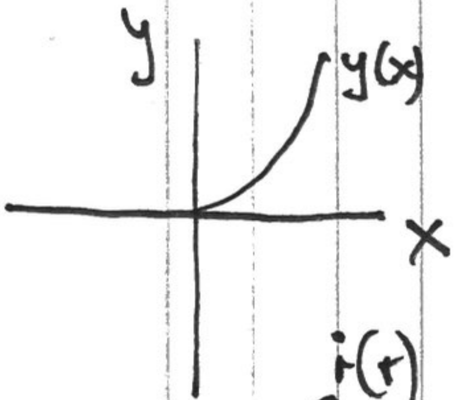
Arnold Sommerfeld

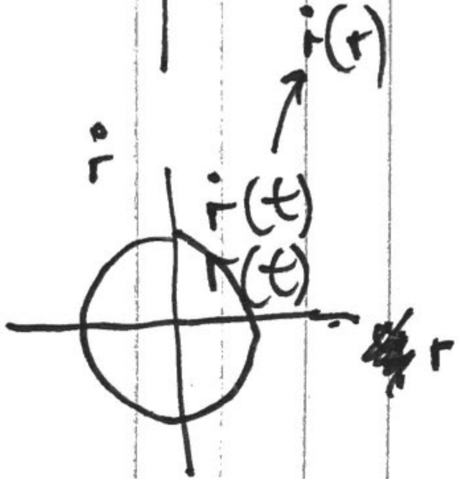
mache Coriolis mit rotierender Stange in Polarkoord.



$$\mathcal{L} = \frac{m}{2} v_\varphi^2 + \frac{m}{2} v_r^2 = \frac{m}{2} r^2 \dot\varphi^2 \equiv \frac{1}{2} r^2 \omega^2 + \frac{1}{2}\dot r^2$$

$$\dot\varphi = \omega = \text{const}$$





$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot r} = \ddot r \,, \quad \frac{\partial \mathcal{L}}{\partial r} = r\omega^2$$

$$\ddot r - r\omega^2 = 0 \,, \quad \ddot r = r\omega^2 = \frac{v_\varphi^2}{r}$$

= Zentrifugal

und v?

Sei q verall. Variable. Dann

$$\frac{\partial \dot q}{\partial q} = \frac{\partial q}{\partial \dot q} = 0$$

sind unabh.

Energiesatz

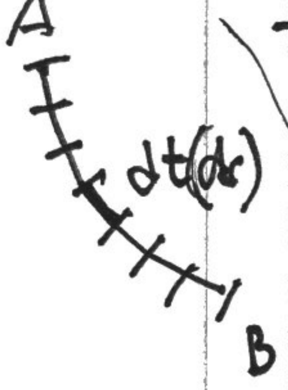
$$\frac{1}{2}\cancel{m}v^2 - \cancel{m}gy = 0$$

$$v = \sqrt{2gy}$$

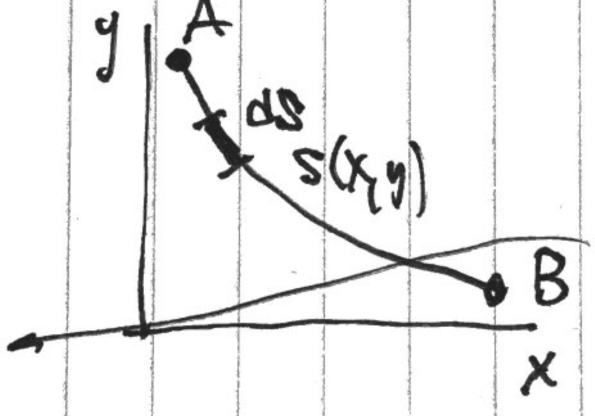
Brachistochrone

schnellste Bahn $y(x)$

$$0 = \delta t$$



$$t_{AB} = \underbrace{\int_{s_A}^{s_B}}_{= \int_{t_A}^{t_B} dt} \frac{ds}{v} = \int \frac{\sqrt{dx^2 + dy^2}}{v} = \int_{x_A}^{x_B} dx \, \underbrace{\frac{\sqrt{1 + y'(x)^2}}{\sqrt{2g\,y(x)}}}_{F(y(x),\, y'(x))}$$



$$\boxed{\text{E-L: } \frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0}$$

$$\frac{d}{dx}\frac{\partial F}{\partial \frac{dy}{dx}}$$

Herleitung E-L

$\mathcal{L}(q)$ $\quad \frac{\partial \mathcal{L}}{\partial q} = 0$

$\frac{dy}{dx} = 0$

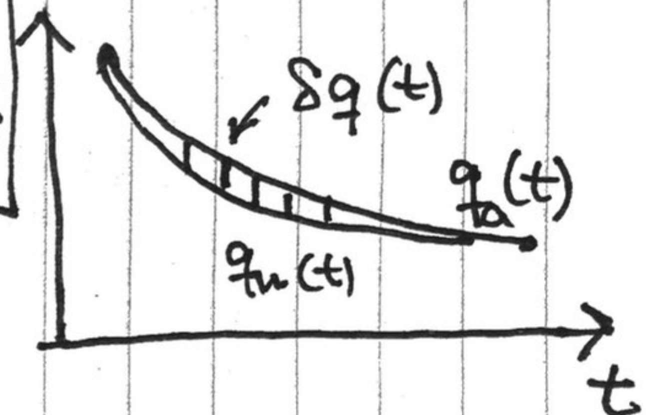
$$0 = \delta S = \delta \int dt\, \mathcal{L}(q, \dot{q})$$

$$\overset{(a)}{=} \int dt\, \delta \mathcal{L}$$

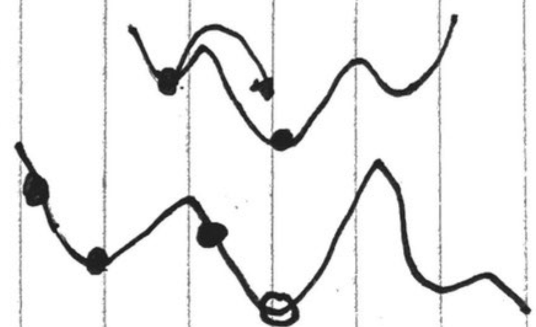
soll so sein

partielle Integration

$$= \int dt \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta \dot{q} \right]$$



$$= \int dt \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta \frac{dq}{dt} \right]$$



$$= \int dt \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot{q}}}_{f(x)} \underbrace{\frac{d}{dt}}_{\frac{d}{dx}} \underbrace{\delta q}_{g(x)} \right]$$