Kap II Newton

2. Gravitation

„2-Körperproblem"

$$m_1 \ddot{\vec{r}}_1 = -G m_1 m_2 \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

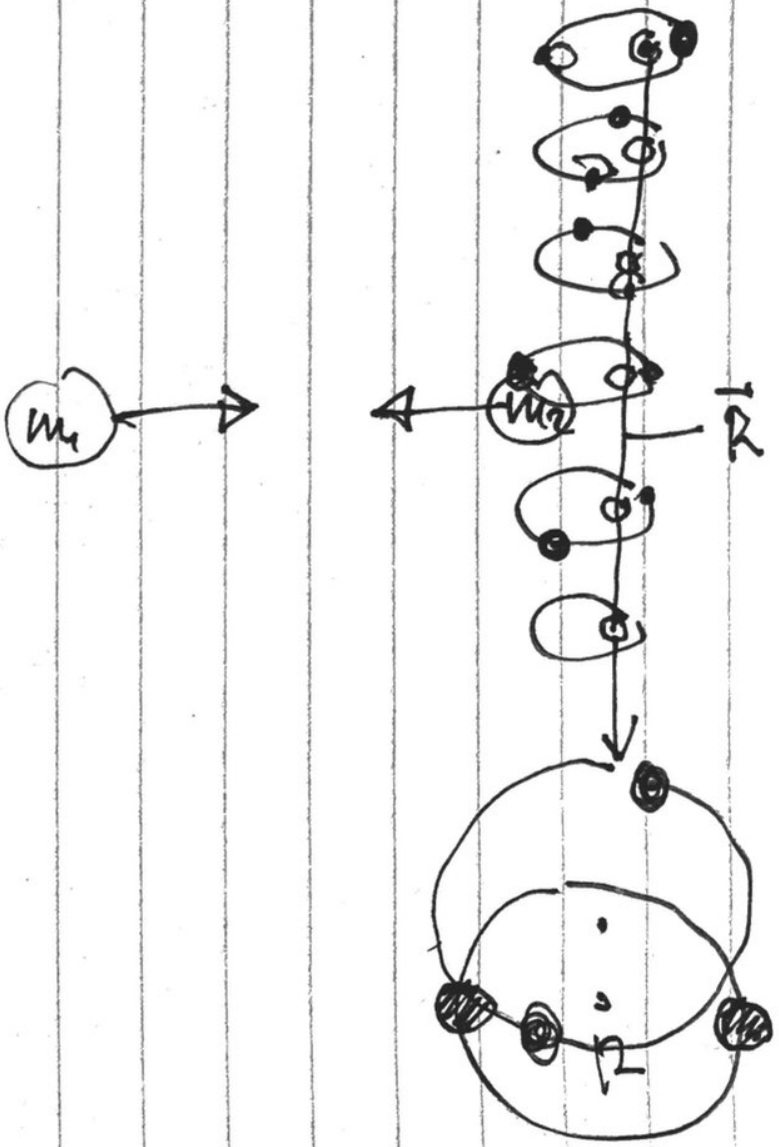
$$m_2 \ddot{\vec{r}}_2 = -G m_1 m_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_1 - \vec{r}_2|^3}$$

addieren $m_1 \ddot{\vec{r}}_1 + m_2 \ddot{\vec{r}}_2 = 0$

oder $\boxed{\ddot{\vec{R}} = 0}$ mit

Schwerpunkt $\vec{R} = \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

Schwerpunkt folgt Galilei



$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = -G \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} \frac{(m_1 + m_2)}{m_1 m_2} \; m_1 m_2 \qquad \Gamma = m_1 m_2 G$$

$$m = \frac{m_1 m_2}{m_1 + m_2} \quad \text{„reduzierte Masse“}$$

$$\boxed{m \ddot{\vec{r}} = -\Gamma \frac{\vec{r}}{r^3}} \quad \text{mit} \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

nichtlineare DGL !! Reihenentwicklung nach Rundman

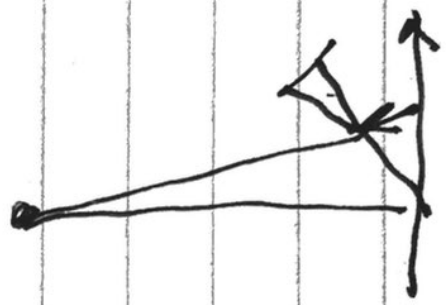
will 0 !! multipl. mit $\dot{\vec{r}}$

$$\boxed{\ddot{\vec{r}} \times \vec{r} = 0} = \dot{\vec{r}} \times \vec{r} = \ddot{\vec{r}} \times \vec{r} + \cancel{\dot{\vec{r}} \times \dot{\vec{r}}}^{\,0}$$

P.R.

$$\vec{r} \times \dot{\vec{r}} = 0$$

Flächensatz



$m\vec{r} \times \dot{\vec{r}} = \text{const}$

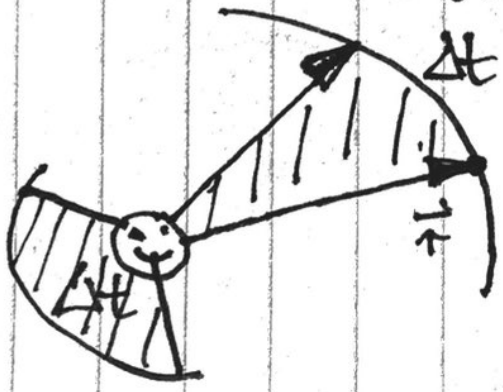
$\vec{r} \times \vec{p} = \vec{L} = \text{const}$

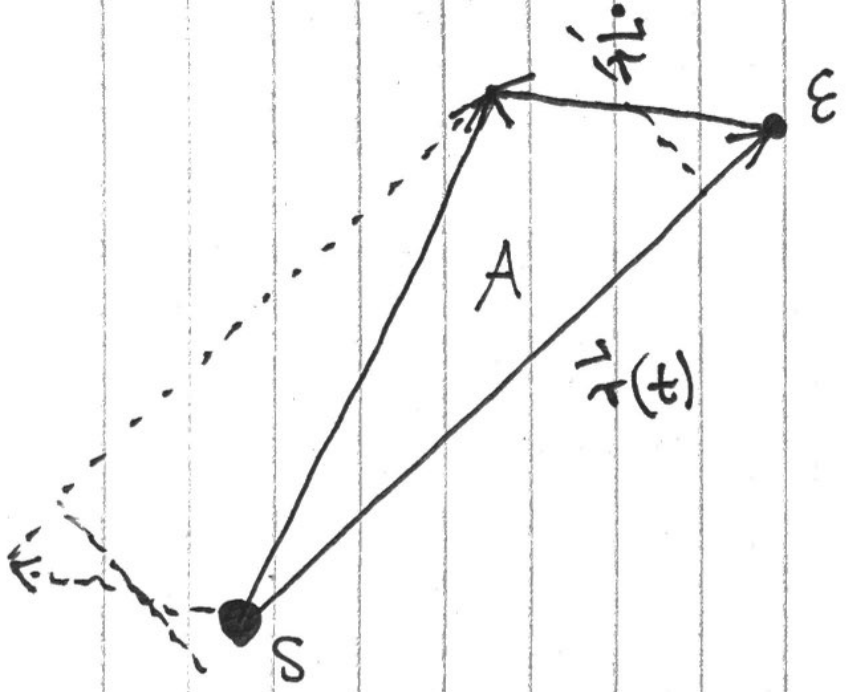
sei $\vec{F}$ Zentralkraft. Zeige $\vec{L} = \text{const}$

$\vec{F} \sim \vec{r}$ usw.

Ü: Zeige $\vec{L} = \vec{L}_1 + \vec{L}_2$

Kepler: Planet – Fahrstrahl überstreicht in gleichen Zeiten gleiche Flächen

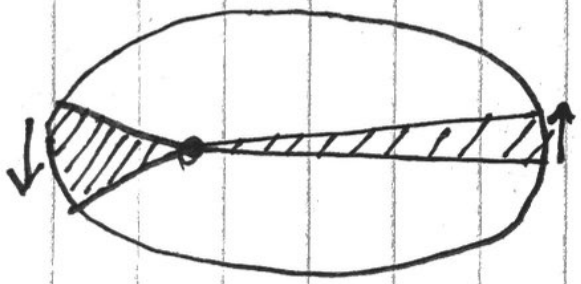


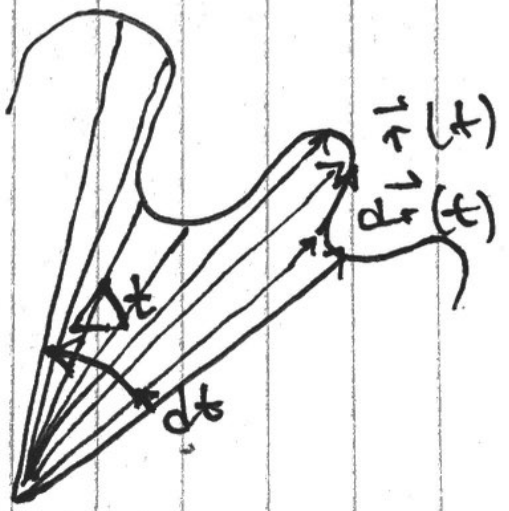


$$\dot{A} = \frac{1}{2} \underbrace{\vec{r} \times \dot{\vec{r}}}_{\text{const}} = \ell$$

$$\underline{A(t)} = \int_0^t dt\, \dot{A} = \underline{\ell t}$$

$$\Delta A(t) = \ell \Delta t$$





Schule