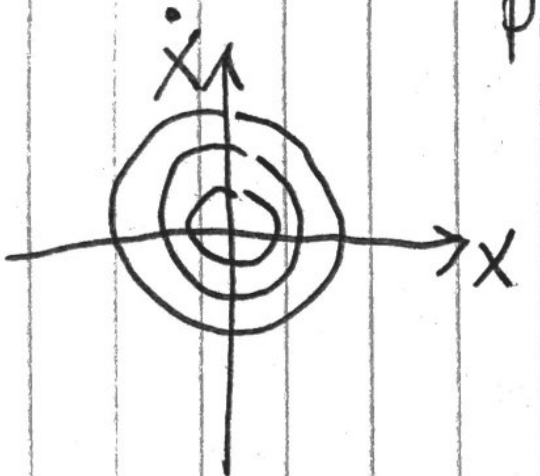


Phasenraum

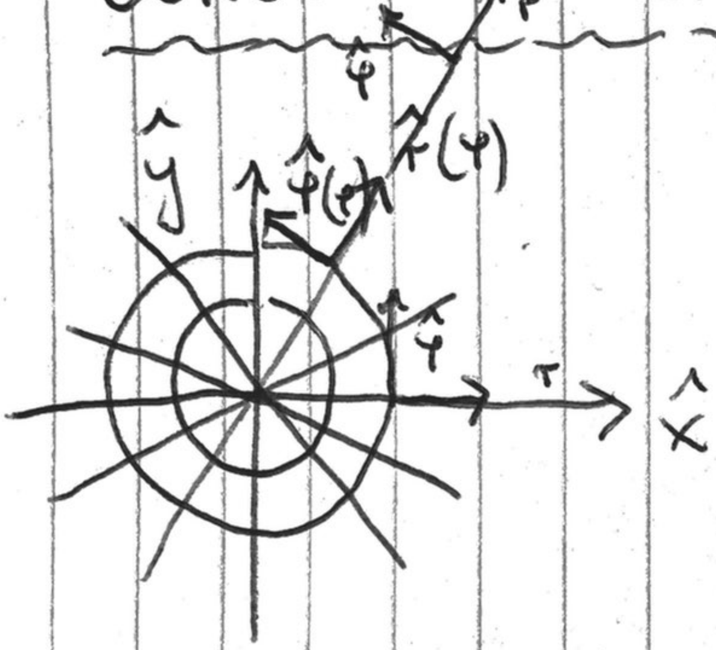
phase space

$$\dot{x}\,\ddot{x} = -x\,\dot{x}$$

$$\frac{1}{2}\,\overset{\cdot}{\widehat{\dot{x}^{2}}} = -\frac{1}{2}\,\overset{\cdot}{\widehat{x^{2}}}$$

$$\underbrace{\frac{1}{2}\dot{x}^{2}}_{T_{kin}} + \underbrace{\frac{1}{2}x^{2}}_{V_{pot}} = E$$

Coriolis & Zentrifugal

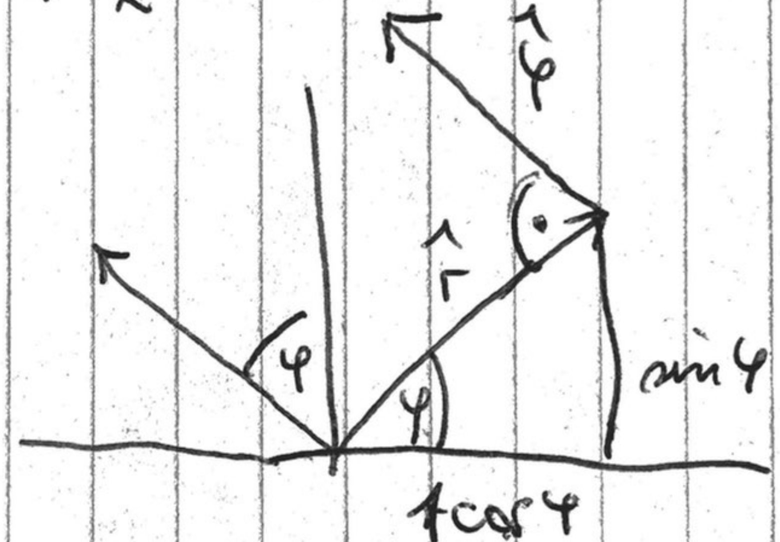


$$\hat{\varphi} = \vec{e}_\varphi$$

$$\hat{r} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

$$\hat{\varphi} = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}$$

$$\hat{r} \cdot \hat{\varphi} = 0$$



$$\boxed{\begin{aligned} \hat{r} &= \hat{x}\cos\varphi + \hat{y}\sin\varphi \\ \hat{\varphi} &= -\hat{x}\sin\varphi + \hat{y}\cos\varphi \end{aligned}}$$

$\vec{v}$ in Pol. Ko.

$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}\right\} \text{Trafo}$$

$$\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y} = \dot{\vec{r}} = v_x\hat{x} + v_y\hat{y}$$

$$= \dot{\overline{r\cos\varphi}}\,\hat{x} + \dot{\overline{r\sin\varphi}}\,\hat{y}$$

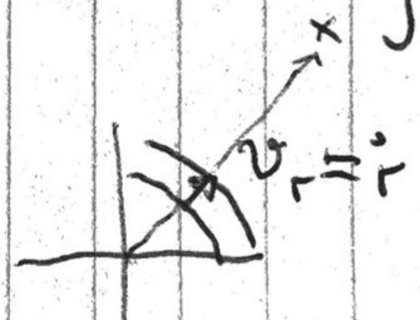
$$\cancel{= \dot{r}(\cos\varphi\hat{x} + \sin\varphi) + r\dot{\varphi}(-\sin\varphi + \cos\varphi)}$$

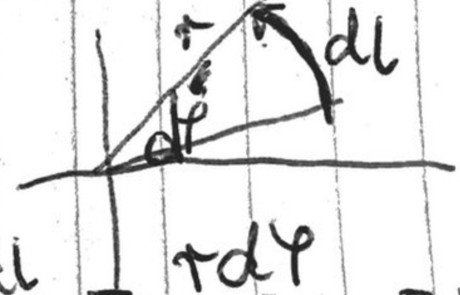
$$= \dot{r}\underbrace{(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y})}_{\hat{r}} + r\dot{\varphi}\underbrace{(-\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y})}_{\hat{\varphi}}$$

$$= \dot{r}\,\hat{r} + r\dot{\varphi}\,\hat{\varphi}$$

$$\overset{!}{=} v_r\,\hat{r} + v_\varphi\,\hat{\varphi}$$

$$\boxed{\begin{aligned} v_r &= \dot{r} \\ v_\varphi &= r\dot{\varphi} \end{aligned}}$$





$$v_\varphi = \frac{dl}{dt} = \frac{r\,d\varphi}{dt} = r\dot{\varphi}$$

sei $\vec{a}$ beliebig

$$\boxed{\begin{aligned} a_r &= \ddot{r} - r\dot{\varphi}^2 \\ a_\varphi &= r\ddot{\varphi} + 2\dot{r}\dot{\varphi} \end{aligned}}$$

$$r\dot{\varphi}^2 = \frac{(r\dot{\varphi})^2}{r} = \frac{v_\varphi^2}{r}$$

$$2\dot{r}\dot{\varphi} = 2 v_r v_\varphi / r$$

$$a_x = \vec{a} \cdot \hat{x}$$

$$= \left(a_r \hat{r} + a_\varphi \hat{\varphi}\right) \cdot \hat{x}$$

$$= \left[a_r \left(\cos\varphi\, \hat{x} + \sin\varphi\, \hat{y}\right) + a_\varphi \left(-\sin\varphi\, \hat{x} + \cos\varphi\, \hat{y}\right)\right] \cdot \hat{x}$$

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$$= a_r \cos\varphi - a_\varphi \sin\varphi$$

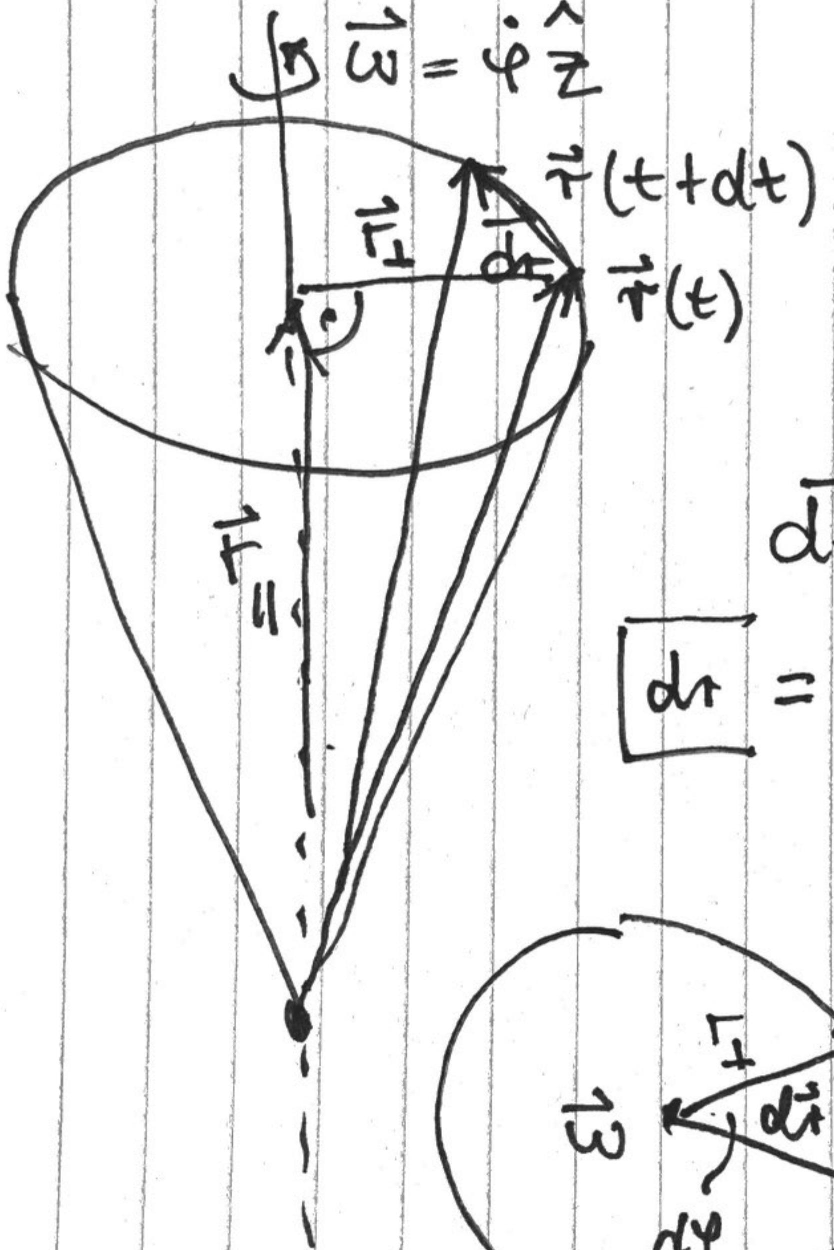
$$a_y = a_r \sin\varphi + a_\varphi \cos\varphi$$

speziell sei $\vec{a}$ Beschleunigung $= \ddot{\vec{r}} = \overset{\bullet\bullet}{\overbrace{\left(\underset{= r\cos\varphi}{x}\, \hat{x} + \underset{= r\sin\varphi}{y}\, \hat{y}\right)}}$

$$a_x = \ddot{x} = \left(\ddot{r} - r\dot{\varphi}^2\right)\cos\varphi - \left(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\right)\sin\varphi$$

$$a_y = \ddot{y} = \left(\ddot{r} - r\dot{\varphi}^2\right)\sin\varphi + \left(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}\right)\cos\varphi$$

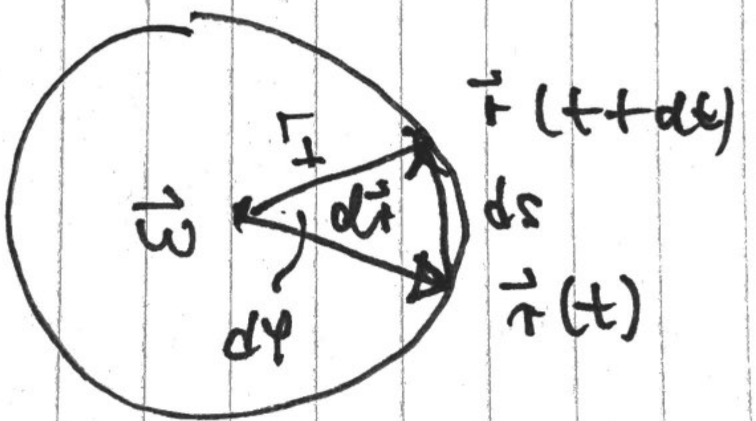
Eulerformel rotierende Vektoren

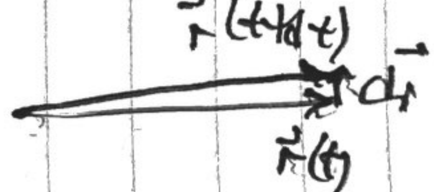


$\vec{r}$ rotiert ~~um~~ mit festem Fußpunkt um $\vec{\omega}$ - Achse

$$\vec{dr} = dr\,\hat{\varphi}$$

$$\boxed{dr} = ds = r_\perp\, d\varphi = \boxed{r_\perp\, \omega\, dt}$$





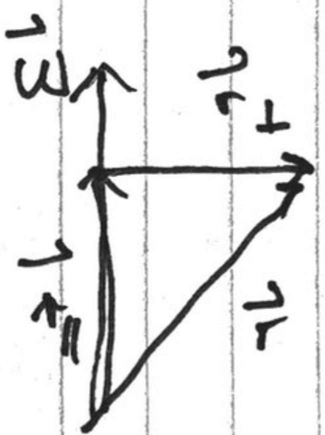
übrigens ist $\vec{\omega}, \vec{r}_\perp, \vec{dr}$ Rechtssystem

also $\hat{dr} = \hat{\omega} \times \hat{r}_\perp$

vgl. $\hat{\varphi} = \hat{z} \times \hat{r}$ in Zylinderkoordinaten

$$dr = r_\perp \, \omega \, dt$$

(malnehmen) $\hat{dr} = \hat{\omega} \times \hat{r}_\perp$



$$\vec{\omega} \times \vec{r}_\parallel = 0$$

$$\vec{dr} = \vec{\omega} \times \vec{r}_\perp \quad dt$$

$$\frac{\vec{dr}}{dt} = \vec{\omega} \times \vec{r}_\perp + \vec{\omega} \times \vec{r}_\parallel = \vec{\omega} \times (\vec{r}_\perp + \vec{r}_\parallel)$$

$$\boxed{\dot{\vec{r}} = \vec{\omega} \times \vec{r}} \qquad \left.\frac{d}{dt}\right|_{rot} = \vec{\omega} \times \_ \;\hat{=}\; \vec{\omega} \times$$