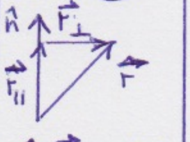
Klausur Theo Mech, 2012

(1) $\nabla\Phi = \frac{\partial\Phi}{\partial n}\hat n$

$\nabla\frac{1}{r} = \frac{\partial(1/r)}{\partial r}\hat r = -\frac{\hat r}{r^2}$

$\nabla z = \frac{\partial z}{\partial z}\hat z = \hat z$

Rest (13)

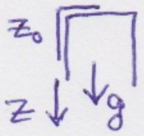


$\hat n\cdot\vec r = r_\parallel$

$r^2 - r_\parallel^2 = r_\perp^2$

wie bekannt

(2)



$\underbrace{ma}_{\text{gesamte Masse wird beschleunigt}} = \underbrace{\frac{z}{L}mg}_{\text{g wirkt nur auf überhäng. Stück}}$

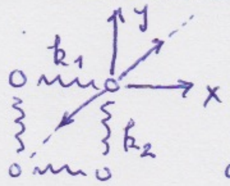
$\ddot z = \frac{z}{L}g$ Ansatz $z = z_0 e^{\alpha t}$

$\alpha^2 z = \dot z\frac{g}{L}$, $\alpha = \sqrt{\frac{g}{L}}$, $z = z_0 e^{t\sqrt{g/L}}$

(3) $\vec r\times\dot{\vec r} = \text{const}$, davon z-Komponente

$x\dot y - y\dot x = \text{const}$

(4)



Reine Längenänderung nur möglich für Bewegung der Massen entlang Diagonale!

$$\left.\begin{aligned} m\ddot x &= -k_1 x + k_2\cdot 0\\ m\ddot y &= k_1 0 - k_2 y\end{aligned}\right\}\ \ddot{\vec r} = -\frac{k}{m}\vec r$$

$\longrightarrow \omega = \sqrt{\frac{k}{m}}$

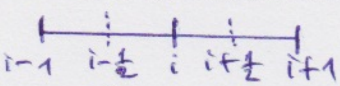
(5) Die inverse Trafo ist bei Drehung $\varphi\to-\varphi$

$$\to\left.\begin{aligned} x &= x'c - y's\\ y &= x's + y'c\\ z &= z'\end{aligned}\right\}\ \begin{aligned}&\text{mit } s = \sin\varphi, c = \cos\varphi\\ &\dot x = \dot x'c - \dot y's - x'\omega s - y'\omega c\\ &\dot y = \dot x's + \dot y'c + x'\omega c - y'\omega s\end{aligned}$$

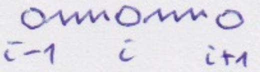
$$\begin{aligned}\dot x^2 + \dot y^2 + \dot z^2 = {} & \dot x'^2c^2 + \dot y'^2s^2 + x'^2\omega^2s^2 + y'^2\omega^2c^2\\ & + \dot x'^2s^2 + \dot y'^2c^2 + x'^2\omega^2c^2 + y'^2\omega^2s^2\\ & + 0 + 0 - \dot x'y'\omega c^2 + 0 + \dot y'x'\omega s^2\\ & - \dot x'y'\omega s^2 + \dot y'x'\omega c^2 + \dot z^2,\ \text{also}\end{aligned}$$

$$\begin{aligned} L &= \frac12 m(\dot x^2 + \dot y^2 + \dot z^2)\\ &= \frac12 m\left[(\dot x'^2 + \dot y'^2 + \dot z'^2) + \omega^2(x'^2 + y'^2) + \omega(x'\dot y' - \dot x'y')\right]\end{aligned}$$

(6)



$$\left.\frac{d^2f}{dx^2}\right|_i = \frac{d}{dx}f'\Big|_i = \frac{f'_{i+1/2} - f'_{i-1/2}}{dx} = \frac{\frac{f_{i+1}-f_i}{dx} - \frac{f_i - f_{i-1}}{dx}}{dx} = \frac{f_{i+1} - 2f_i + f_{i-1}}{dx^2}$$



$$\begin{aligned} m\ddot\xi_i &= -k(\xi_i - \xi_{i-1})\\ &\quad + k(\xi_{i+1} - \xi_i)\end{aligned}$$

$m\ddot\xi_i = k(\xi_{i+1} - 2\xi_i + \xi_{i-1}) = k\xi_i''dx^2$

$\cancel{m}\left(\frac{d^2}{dt^2} - \frac{k}{m}dx^2\frac{d^2}{dx^2}\right)\xi_i = 0$

$\left(\frac{d^2}{dt^2} - \omega^2dx^2\frac{d^2}{dx^2}\right)\xi_i = 0$

$\left(\frac{d^2}{dt^2} - a^2\frac{d^2}{dx^2}\right)\xi(x,t) = 0$

(7) $0 = \delta S = \delta\int\sqrt{dx^2 + dy^2} = \delta\int dx\sqrt{1 + y'^2}$

E-L: $0 = \frac{d}{dx}\frac{\partial\sqrt{1+y'^2}}{\partial y'} - \frac{\partial\sqrt{1+y'^2}}{\partial y}$

$0 = \frac{d}{dx}\frac{\cancel{2}y'}{\cancel{2}\sqrt{1+y'^2}} = \frac{\sqrt{1+y'^2}\,y'' - y'\frac{2y'y''}{2\sqrt{1+y'^2}}}{1+y'^2}$

$= \frac{(1+y'^2)y'' - y'^2y''}{1+y'^2} = \frac{y''}{1+y'^2}$

d.h. $y'' = 0 \to y = ax + b$, Gerade

(8) $z = z_0 + \frac{p_0}{m}t + \frac12 gt^2$

$p = p_0 + mgt$

$\to z_0 = z - \frac{p_0}{m}t - \frac12 gt^2$

$= z - \left(\frac{p - mgt}{m}\right)t - \frac12 gt^2$

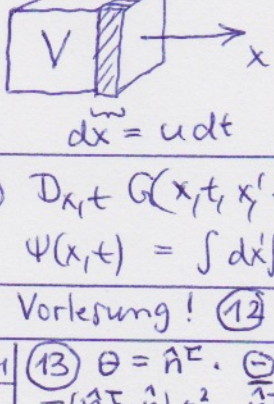
$z_0 = z - \frac{p}{m}t + \frac12 gt^2 = z_0(z,p)$

$p_0 = p - mgt \qquad = p_0(z,p)$

$\{z_0,p_0\}_{z,p} = \frac{\partial z_0}{\partial z}\frac{\partial p_0}{\partial p} - \frac{\partial z_0}{\partial p}\frac{\partial p_0}{\partial z}$

$= 1\cdot 1 - \frac{t}{m}\cdot 0 = 1.$ D.h. $(z_0,p_0)\to(z,p)$ ist eine kanon. Trafo, weil (z_0,p_0) und (z,p) kanon. Variable sind. D.h. zeitliche Entwicklung ist eine kanon. Transf.

(9) $\frac{dV}{dt} = \frac{d}{dt}\int_V dx\,dy\,dz = \oint_{\partial V}dx\,dy\,w + \oint_{\partial V}dx\,dz\,v + \oint_{\partial V}dy\,dz\,u$



$dx = u\,dt$

(10) $D_{x,t}G(x,t,x',t') = \overbrace{\delta(x-x')\,\delta(t-t')}^{\text{neu}}$

$\Psi(x,t) = \int dx'\int dt'\,G(x,t,x',t')\,f(x',t')$

(11) Vorlesung! (12) $\underline{\Theta} = (\Theta_i\delta_{ij})$ mit Trägheitsmoment Θ_i

(13) $\Theta = \hat n^T\cdot\underline{\Theta}\cdot\hat n = \hat n^T\cdot\int\rho\,dV(r^2\mathbb{1} - \vec r\vec r^T)\cdot\hat n$

$= (\hat n^T\cdot\hat n)r^2 - \hat n^T(\vec r\vec r^T)\hat n = r^2 - (\hat n\cdot\vec r)^2$