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Instability, Runaway and X-rays

Achim Feldmeier

Imperial College, Prince Consort Road, London SW7 2BZ, UK
email: a.feldmeier@ic.ac.uk

Abstract. The first part discusses X-ray emission from hot star winds, in the framework of the line-driven instability. The latter causes strong shocks to form in the wind, which enclose shells of highly compressed gas. Wind turbulence can ablate cloudlets from dense gas ahead of the shells, which are then accelerated through void regions until they collide with the next outer shell. The corresponding X-ray flashes can match observed fluxes.

In the second part, I discuss why (and whether) line-driven winds adopt a unique, critical solution. Abbott waves propagate inwards from everywhere along a shallow solution. Perturbations introducing negative velocity gradients can perform work on the wind, and hence accelerate the gas. If these perturbations are located below the critical point, the runaway does not stop at the CAK solution, but leads to overloaded winds. Furthermore, if Abbott waves are not accounted for in the Courant time step and/or boundary conditions, numerical runaway results.

1. Introduction

Line-driven winds show a rich variety of flow phenomena. The present article discusses two effects which play a role even for the simplest flow geometries, i.e., planar or spherical winds from single stars, avoiding thereby complexities of wind-compressed disks, co-rotating interaction regions, accretion disk winds, colliding winds, high-mass X-ray binaries, ablation-driven winds, etc. These are the X-ray emission from O stars and the evolution of line-driven winds towards a unique, critical solution. It is suggested that both phenomena are interrelated, with X-rays being due to second-order unstable Abbott waves, and overall flow evolution due to first-order runaway Abbott waves.

2. X-ray emission

Owocki et al. (1988) were the first to calculate the hydrodynamic wind structure which develops from the line-driven instability. These simulations show that the wind consists of a train of strong reverse shocks, which decelerate thin, fast gas and compress it into dense shells. Hillier et al. (1993) challenged the long-held opinion that the line-driven instability should be responsible for X-ray emission from hot star winds. From the above, time-dependent wind models, they estimated that hot post-shock gas is too thin to cause any significant X-ray emission. This gas should only account for 1% to 10% of the observed X-rays as inferred from Einstein or ROSAT data.

Instead of continuous X-ray emission from thin gas which is fed steadily through shocks, Feldmeier et al. (1997) suggested that X-rays are created in flashes, when fast, tiny wind cloudlets collide with the instability-generated shells. The small-scale cloudlets may be able to explain both the X-ray luminosity, and the approximate constancy of the X-ray flux. While thus far the modeling has assumed a spherically symmetric, purely radial wind structure, and leads to major variability in X-ray fluxes, it is strongly expected on physical grounds that cloudlets have a small lateral extent.

The physical difference between the cloudlets and shells is as follows. The shells start to form out of *homogeneous* gas near the base of the wind. Momentum gain from stellar photons preferentially increases the mass of a shell, not its speed: any extra acceleration pushes the shell into outer gas, which is swept up, decelerating the shell to stationary wind speed. In contrast, cloudlets form at the rim of dense gas ahead of a shell in a kind of ablation process. Since almost no gas lies ahead of a cloudlet, the latter can be accelerated to high speeds.

Figure 1 shows a snapshot of the calculated wind structure of the O9 I star ζ Ori. Essentially all X-ray emission stems from one single shock slightly above $6\,R_*$. For the corresponding X-ray spectrum, which fits ROSAT observations rather well, and the relation of the X-ray emission site to a cloudlet-shell collision, we refer to Figures 8 and 17 in Feldmeier et al. (1997).

A large flux of cloudlets is required to generate significant X-ray emission. To create cloudlets in abundance, stochastic perturbations simulating photospheric turbulence were applied at the inner boundary. The perturbation amplitude is 30% of the sound speed, which is below the limit of the measured turbulent velocity dispersion in hot star atmospheres (Conti & Ebbets 1977). During their passage through the exponential density stratification, perturbations grow into shocks. These shocks are not related to the line-driven instability; instead, they act as seed perturbations for the instability. The assumed photospheric perturbations are strong enough that atmospheric shocks are largely saturated, hence the X-ray luminosity of the wind is essentially independent of base perturbation amplitudes.

While the above scenario gives promising results in the one case studied so far (ζ Ori), future work has to focus on general aspects of stellar X-ray emission. We need to understand:

(1) the constancy of the observed X-ray emission. This requires extension of current wind hydrodynamic models to 2-D. A simple estimate shows that $\leq 10^4$ independent radial wind rays, each with its own cloudlet-shell collisions taking place, could explain the observed flux constancy due to angle averaging. This number of independent wind rays is well below the number of expected coherent turbulent cells in the photosphere, when one identifies the turbulent coherence time with the sound crossing time of a cell.

(2) the universal scaling $L_{\rm X}/L_{\rm bol} \sim 10^{-7}$ for O stars, and why it breaks down for near-main-sequence B stars (Cohen et al. 1997) and for Wolf-Rayet stars (Wessolowski 1996). Such a general scaling is hard to understand for wind embedded sources, where emergent X-ray fluxes depend on a delicate balance between emission and absorption (Owocki & Cohen 1999).

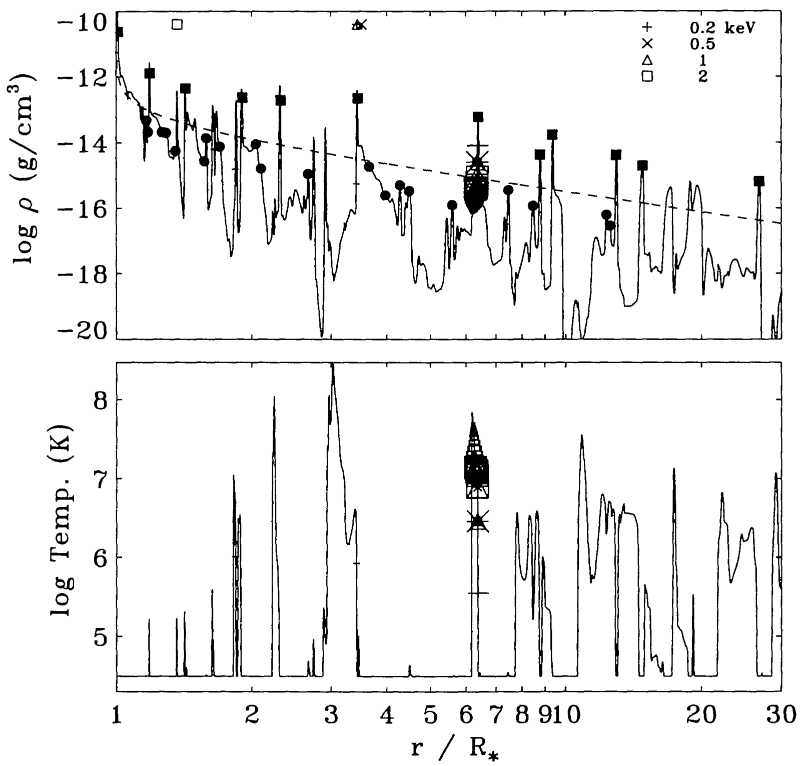

Figure 1. Snapshot of density and temperature run for a wind model of ζ Ori. Filled bullets mark fast cloudlets, filled squares mark dense shells. Symbols ($+$, $\times$, etc) indicate strong X-ray emission at different energies, the symbol size scaling with emissivity. At the upper margin, locations of the X-ray photosphere ($\tau = 1$) at these energies are indicated.

3. Runaway towards a critical solution

Our question in this section is, why line-driven winds from stars and accretion disks adopt a critical solution out of an infinite number of possible solutions.

The stationary Euler equation for a 1-D, planar wind at zero sound speed, which is subject to a constant radiative flux and a Sobolev-type line force, is

$$vv' = -g + C(vv')^{\alpha}, \tag{1}$$

with velocity v, $v' = dv/dr$, gravity g, and CAK parameter $0 < \alpha < 1$ (Castor, Abbott, & Klein 1975). The constant C decreases with increasing mass loss rate. Below a certain, limiting mass loss rate, two solutions vv' exist for any value of C, termed shallow and steep. At the maximum or CAK mass loss rate, a shallow and steep solution merge in a continuous and differentiable fashion at a critical point.

At zero sound speed, both solution families are globally defined, as is easily seen from solving eq. (1) for $\alpha = 1/2$. For finite gas pressure, however, CAK showed that steep solutions do not extend into the subsonic regime including the photosphere, whereas shallow solutions do not reach infinity (for a spherically symmetric wind). Hence, the true wind solution should be the critical one, which switches from a shallow to a steep solution at the critical point.

There are three arguments against this reasoning. (1) It seems odd that the gas pressure should determine the wind solution. There is no fundamental, physical argument against winds at zero sound speed. (2) The termination of shallow solutions at large radii is due to the fact that they cannot perform the required expansion work. The wind then starts to decelerate – which was excluded by CAK. However, since deceleration occurs only at speeds much larger than the local escape speed, the wind still reaches infinity at positive speed. Hence, shallow + decelerating solutions are globally defined. (3) In time-dependent hydrodynamic simulations, the outer boundary is usually chosen far below the termination radius of shallow solutions. Over the whole calculational grid, the latter therefore exist and increase monotonically in velocity, and the CAK argument cannot explain why the code would converge to the critical solution.

I suggest in the following that the evolution of shallow solutions towards the CAK solution is due to communication and feedback of the outer wind and the photosphere (where the mass loss rate is fixed), as brought about by Abbott waves.

Abbott waves are the fundamental wave mode for line-driven winds, much as sound waves are for ordinary flows, and Alfvén waves are for hydromagnetic flows. The characteristic form of the Euler equation for a planar wind at zero pressure, assuming a line-force exponent $\alpha = 1/2$, is

$$\left[\frac{\partial}{\partial t} + v\left(1 \mp \frac{1}{q_\pm}\right)\frac{\partial}{\partial x}\right]\frac{v'}{\rho} + \frac{1}{\rho}\frac{\partial g}{\partial x} = 0. \tag{2}$$

The upper signs apply for $v' > 0$, the lower signs for $v' < 0$. A line force $\propto \sqrt{|v'|/\rho}$ was assumed. The dimensionless parameter q is given by

$$q_\pm = \sqrt{\pm w\, m}, \tag{3}$$

where $w = vv'/v_c v'_c$ and $m = \rho v/\rho_c v_c$ are the acceleration and mass loss rate in units of their CAK values, respectively. Simple algebra shows that eq. (2) is equivalent to the time-dependent generalization of eq. (1), when the latter is differentiated with respect to x in order to bring it into quasi-linear form.

In the WKB approximation, g' can be neglected in eq. (2). Hence, for $v' > 0$, a wave of amplitude v'/ρ propagates inwards, in the stellar rest frame, at a phase speed $(1 - 1/q)v < 0$, if $w < 1$ and $m < 1$. The latter two conditions define precisely a shallow solution. It follows that shallow solutions are sub-Abbottic. They are therefore analogous to solar wind breezes. Furthermore, at the critical point, $q = 1$, and Abbott waves stagnate (Abbott 1980). Note, however, that in contrast to the sound or Alfvén speed there is no physical meaning to an Abbott speed. Instead, only the product vv' becomes critical. Namely, while for thermal and hydromagnetic winds the critical point is a saddle point in the (r, v) plane, it is a saddle point in the (r, vv') plane for line-driven winds, as follows from eq. (1) (Bjorkman 1995; Feldmeier & Shlosman 1999). This explains the somewhat counter-intuitive fact that shallow solutions can, at large radii, reach velocities larger than the CAK critical point velocity.

Since Abbott waves define the characteristics of line-driven winds, they have to be included in the Courant time step and the boundary conditions (b. c. hereafter). Both was not the case in simulations so far. Instead, pure outflow b. c. were generally assumed. The latter are correct only for the critical solution (and for steep and overloaded solutions, see below), which is super-Abbottic above the critical point. Applying outflow b. c. may cause a systematic drift of the wind towards the critical solution, for which these b. c. indeed hold. While pure outflow b. c. may seem natural for a flow into a vacuum, they prevent to study those dynamical effects which may actually be responsible for driving the wind towards the critical state, by causing instead violent numerical runaway.

Even more obviously, the neglect of Abbott waves in the Courant time step causes numerical runaway, which proceeds usually (however, not necessarily) towards the critical solution.

Numerical runaway is shown in Figure 2, for an initially shallow solution with 80% percent CAK mass loss rate. The latter evolves quickly towards the critical solution. From the figure it appears that this is a boundary effect which propagates inwards.

Using the characteristic equation (2), one can formulate *non-reflecting* b. c. Due to the interpretation of v' as a new, fundamental hydrodynamic variable (in order to make the Euler equation quasi-linear), non-reflecting b. c. are not completely straightforward to formulate, and some arbitrary choices have to be made in the numerical scheme. Details are given elsewhere (Feldmeier & Shlosman 2000b).

Applying non-reflecting b. c. and using a time step which limits the propagation of Abbott waves over the grid, shallow solutions are found to be *mathematically* stable. One may criticize that non-reflecting b. c. stabilize shallow solutions in a *physically* artificial manner, since they annihilate incoming Abbott waves and thereby suppress information exchange with the inner wind which may be responsible for an evolution away from the shallow solution (Owocki, priv. comm.). Bearing this in mind, we note that non-reflecting b. c., in suppressing violent runaway due to boundary inconsistencies, actually *allow* to search for and study

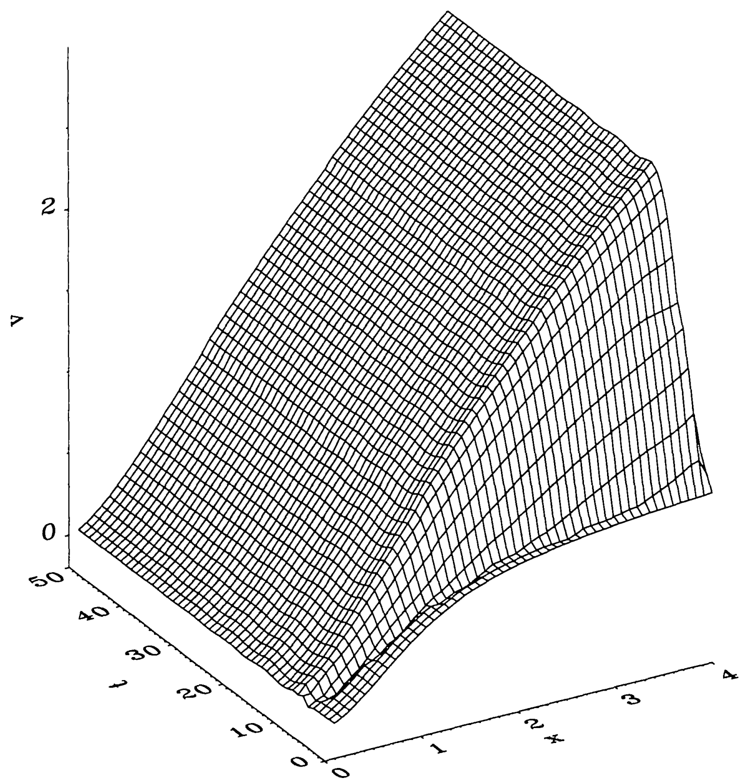

Figure 2. Velocity runaway of a shallow solution towards the critical solution, as caused by the neglect of Abbott waves in the Courant time step and by assuming pure outflow boundary conditions.

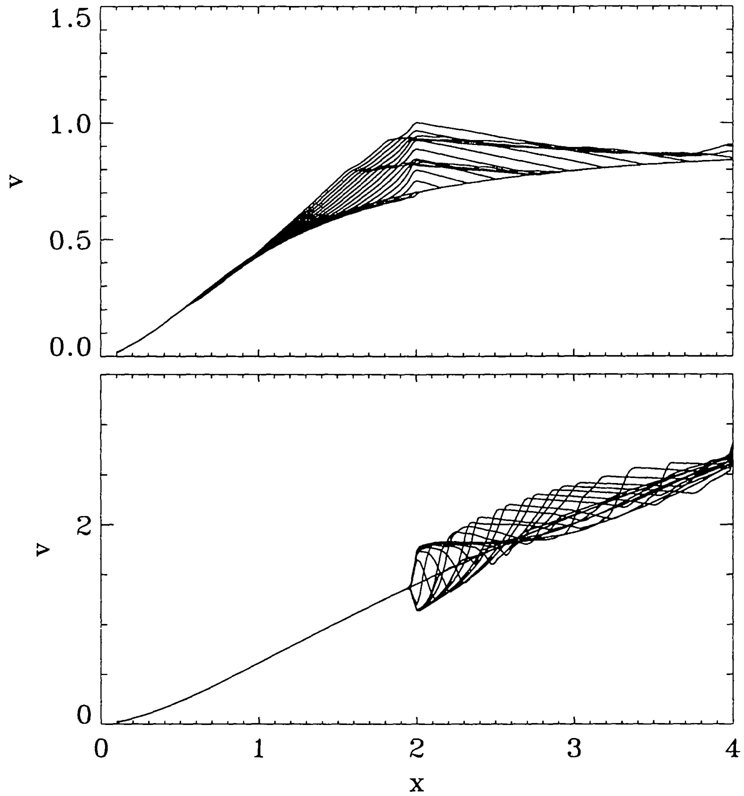

Figure 3. *Upper panel:* Velocity runaway of a shallow solution towards the critical solution, as caused by perturbations which introduce negative velocity slopes. *Lower panel:* Stability of the critical CAK solution.

such new dynamical effects, namely by applying *controlled* perturbations on the wind at interior grid points. We turn to this now.

Equation (2) shows that for negative velocity gradients, the phase propagation is *outwards* directed, with phase speed $(1 + 1/q)v$ (always $q > 0$). This implies a very unique behavior for Abbott waves with alternating signs of v': positive slopes propagate to smaller x, whereas negative slopes propagate to larger x. Hence, the overall wave pattern evolves kinematically towards larger velocities, towards the critical solution.

The upper panel of Figure 3 shows the runaway in a numerical simulation where a temporal sawtooth perturbation was applied at all simulation times, at a fixed position $x = 2$ in the wind. The lower panel shows the unconditional stability of the critical solution.

The velocity runaway can also be understood without reference to Abbott waves, as a direct consequence of the asymmetry of the line force, $g_l \propto (|v'|/\rho)^\alpha$, with respect to positive and negative perturbations in v'. For sufficiently large $|\delta v'|$, positive line force perturbations during $+\delta v'$ cannot be compensated for by negative line force perturbations during $-\delta v'$. Hence, work is performed over an oscillation cycle, and true acceleration of wind gas occurs. Note that, due to $\alpha < 1$, v' has to drop below a certain finite, negative value to cause runaway. Note furthermore that the mediation of the runaway to the inner wind is a true wave property.

Hence, for perturbations being located *above* the critical point, the runaway has to stop at the critical solution, since Abbott waves cannot further propagate inwards once the latter is reached. For these perturbations the CAK solution is therefore the natural outcome of the present runaway.

If, however, perturbations are located *below* the critical point, communication with the wind base is still possible after the wind becomes critical, simply because the region below the critical point is sub-abbottic. The runaway proceeds to a steeper velocity law, until the point at which the perturbation occurs can no longer communicate upstreams. It is readily shown that this corresponds to a *termination* point, at which the velocity law becomes imaginary (Feldmeier & Shlosman 2000a). The wind is then overloaded, i.e., carries a mass loss rate larger than the CAK value. In a neighborhood of the critical point, which defines the bottleneck of the flow, overloaded winds have to stall, causing imaginary v. The winds jump here to the decelerating branch, $v' < 0$, on which it passes through the bottleneck. At some larger height, the radiation field is again able to lift the overloaded wind out of the gravitational potential, hence the solution jumps back to the accelerating branch. In Sobolev approximation, kinks occur in the velocity law at the location of such jumps across solution branches.

This does not yet cover the whole solution manifold. Instead of stationary overloaded solutions, time-dependent overloaded solutions evolve if perturbations occur below a certain radius in the wind (below the critical point). The wind is then so heavily overloaded (still, with a mass loss rate only larger by a few percent than the CAK value) that during deceleration it reaches negative velocities. This inwards falling gas collides with the upwards streaming wind, and within each perturbation cycle strong shocks and accompanying dense shells develop.

We conclude that the CAK critical solution is not a general dynamical attractor. Instead, if sufficiently strong perturbations occur below the critical point, the wind can adopt an overloaded solution.

Finally, we remark that the present runaway due to negative velocity gradients is not related to the line-driven instability discussed in the first part of this paper. The latter occurs only in second order Sobolev theory, when curvature terms v'' are included (Feldmeier 1998). The present runaway is a first order effect.

To summarize this section, we have discussed that:
(1) The reasoning of CAK to rule out shallow wind solutions is too restrictive.
(2) Shallow solutions are sub-abbottic. Abbott waves propagate inwards along them, possibly at large speed.
(3) If this fact is not accounted for in the Courant time step and/or outer boundary conditions, numerical runaway towards the critical solution can occur.
(4) Yet another, physical runaway occurs due to the asymmetry of the line force with respect to positive and negative velocity gradients.

(5) While this runaway, too, drives shallow solutions towards the critical solution, the latter is not a general dynamical attractor. Instead, the wind may adopt overloaded solutions.

Future work has to clarify whether real line-driven winds can be overloaded. Recent suggestions in this direction are by Howk et al. (1999), Porter (this meeting), and Cohen (priv. comm.). Decelerating flow regions should be detectable in P Cygni line profiles, and the shocks in time-dependent overloaded solutions may contribute to the observed X-ray emission, besides the shocks from the line-driven instability.

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Discussion

D. Cohen: Could you elaborate on the causes of the cloudlet ablation? I'm surprised that lower boundary conditions can trigger this relatively far out in the flow.

A. Feldmeier: Physically, one would expect that cloudlet ablation is caused by wind turbulence. Since it is hard to model true, inherent wind turbulence, I simply have chosen to introduce stochastic (or turbulent) photospheric perturbations, which cause in turn stochasticity in the wind.

R. Walder: Can you suggest a physical mechanism which can excite your finite perturbations necessary to drive shallow solutions to the critical one?

A. Feldmeier: The most promising candidates for finite perturbations seem to me wind structure from the line-driven instability and co-rotating interaction regions.

H. Lamers: You predict that if the mass loss rate is too high (overloaded wind), the radiative acceleration may be insufficient at large distances, and the wind may decelerate (you called this a kink in the velocity law). There may be observational evidence for this in the spectra of LBVs. LBVs have high mass loss rates. Several of them, e.g. P Cygni and R81, show an extremely stable absorption component, which is constant over at least a decade. These components seem to occur at a velocity smaller than the edge velocity (Lamers et al. 1985 AA 149, 29 for P Cygni and Wolf et al. for several other LBVs). These components may be the result of either shells ejected long ago, or of a velocity decrease at very large distance.

S. Owocki: First, as comment to Henny's question, the existence of kinks is something that has been seen for a long time. Jean-Paul Koninx's thesis, for example, showed how a standing kink would form in a rapidly rotating wind model. Also, kinks ahead of dense CIRs are the cause of the synthetic DACs in the thesis by Steve Cranmer. But Achim's work is really remarkable in showing the generality and richness of the phenomenon.

Achim, regarding your slow accelerating solutions and choice of right boundary conditions: as you point out, these are analogs to breeze solutions in the solar wind. In that case, they lead to a finite asymptotic pressure, which generally does not match the pressure of the ISM. In the case of CAK winds, it seems your choice of right boundary would likewise have to be quite special to pick out a particular sub-critical solution.

A. Feldmeier: Agreed, but at present it is not clear to me what, in a line-driven wind, would be the analog to gas pressure. It is also true that non-reflecting boundary conditions have a tendency to maintain a particular shallow solution, since they annihilate inward propagating Abbott waves which could otherwise communicate information upstreams about a boundary mismatch.